3.1 Functions

I. Determine whether a relation represents a function.

Definitions

1) Relation: a correspondence (connection) between elements of two sets.

→ a set of ordered pairs.

x has a relation w/ y : $x \longrightarrow y$

Ways to represent relations.

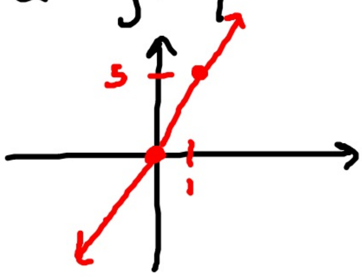
① an equation

$$y = 5x$$

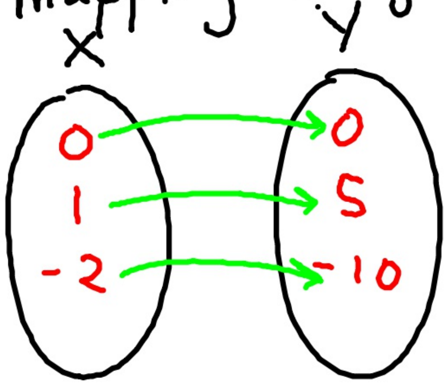
② table

x	y
0	0
1	5
−2	−10

③ a graph



④ mapping diagram



② **Domain** of a relation:

- the set of first elements in a relation
- set of **all** x-values in a relation

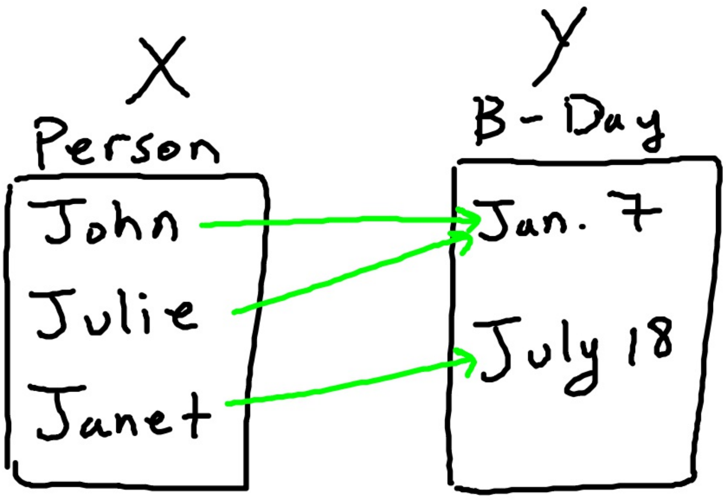
③ **range** of a relation:

- set of of second elements in a relation
- set of **all** y-values in a relation.

$$(\overset{\text{Domain}}{x}, \overset{\text{Range}}{y})$$

Ex:

①



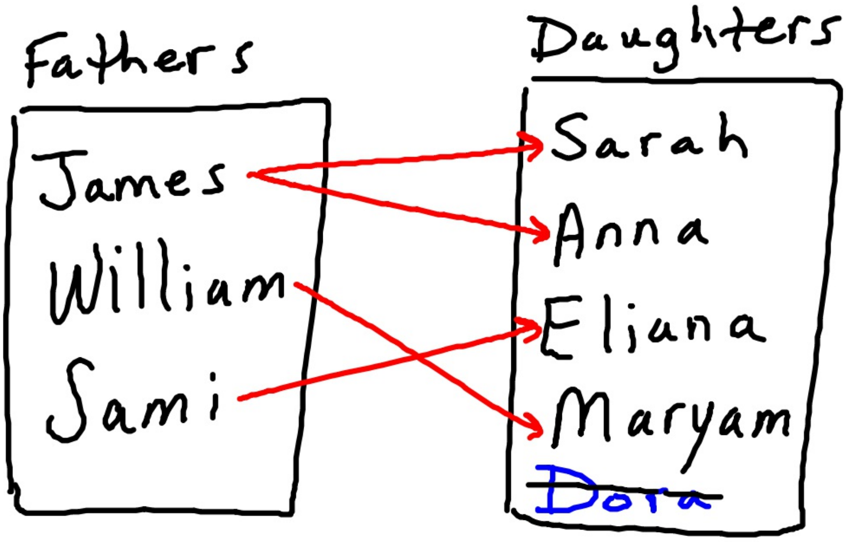
Function

(John, Jan 7), (Julie, Jan 7), (Janet, July 18)

Domain: {John, Julie, Janet}

Range: {Jan 7, July 18}

2



Not a function

D: {James, William, Sami}

R: {Sarah, Anna, Eliana, Maryam}

Function:

a relation that associates with each element in the first set exactly one element in the second set.

Ex: Determine whether the relation represents a function.

a) $\{(-2,4), (0,0), (2,4), (\frac{1}{2}, \frac{1}{4}), (0.5, 2.25)\}$

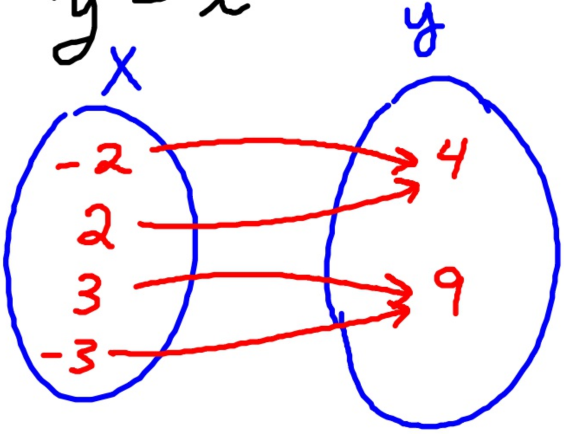
Not a function.

$\frac{1}{2} \to \frac{1}{4}$

$\frac{1}{2} \to 2.25$

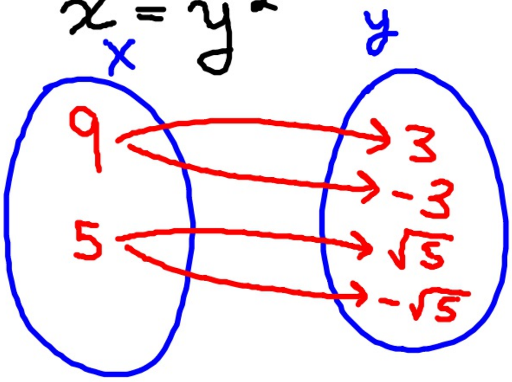
If x's repeat then it's not a function

b) $y = x^2$



Function.

c) $x = y^2$



Not a function.

$9 = y^2$

$y = \pm 3$

$5 = y^2$

$y = \pm\sqrt{5}$

For equations, if y is raised to an even power then it will not be a function.

$$x^2 + y^2 = 16$$

II. Find value of a function.

Note:

① x is an independent variable

② y is a dependent variable

y depends on x

y is $\underbrace{\text{a function of}}$ x

$$\underbrace{y}_{\text{output}} = f(\underbrace{x}_{\text{input}})$$

value of function

argument of function

Ex: For the function f defined by $f(x) = 3x^2 + 2x - 4$, evaluate

① $f(0)$

$$f(0) = 3(0)^2 + 2(0) - 4$$

$$\boxed{f(0) = -4} \longrightarrow (0, -4) \rightarrow \text{y-axis}$$

② $f(-1)$

$$f(-1) = 3(-1)^2 + 2(-1) - 4$$

$$\boxed{f(-1) = -3}$$

③ $f(-x)$

$$f(-x) = 3(-x)^2 + 2(-x) - 4$$

$$\boxed{f(-x) = 3x^2 - 2x - 4}$$

④ $-f(x)$

$$-f(x) = -(3x^2 + 2x - 4)$$

$$\boxed{-f(x) = -3x^2 - 2x + 4}$$

⑤ Evaluate $f(x+h)$

$$f(x+h) = 3\underbrace{(x+h)^2} + 2(x+h) - 4$$

$$= 3\underbrace{(x+h)(x+h)}_{\text{FOIL}} + 2x + 2h - 4$$

$$= 3(x^2 + 2xh + h^2) + 2x + 2h - 4$$

$$\boxed{f(x+h) = 3x^2 + 6xh + 3h^2 + 2x + 2h - 4}$$

⑥ $f(x) + h$

$$\boxed{f(x) + h = 3x^2 + 2x - 4 + h}$$

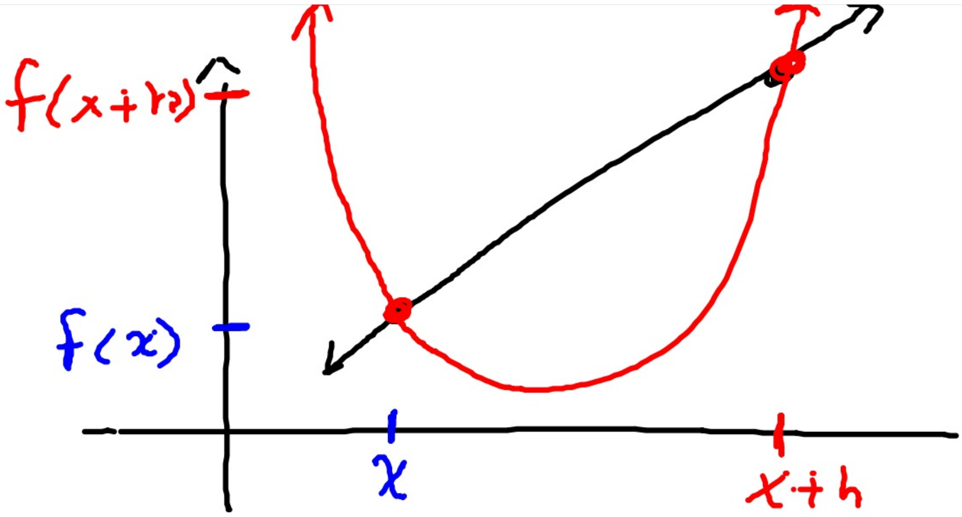
⑦ $\dfrac{f(x+h) - f(x)}{h}$

$$\frac{(3x^2+6xh+3h^2+2x+2h-4)-(3x^2+2x-4)}{h}$$

$$\frac{\cancel{3x^2}+6xh+3h^2+\cancel{2x}+2h-\cancel{4}\cancel{-3x^2}\cancel{-2x}\cancel{+4}}{h}$$

$$\frac{6xh+3h^2+2h}{h} \Rightarrow \frac{6x\cancel{h}}{\cancel{h}}+\frac{3h^{\cancel{2}}}{\cancel{h}}+\frac{2\cancel{h}}{\cancel{h}}$$

$$\boxed{\frac{f(x+h)-f(x)}{h}=6x+3h+2}$$



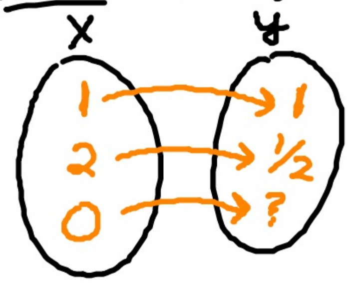
slope of line : $\dfrac{f(x+h) - f(x)}{(\cancel{x}+h) - (\cancel{x})}$

Difference Quotient : $\dfrac{f(x+h) - f(x)}{h}$

III. Finding domains of a function

① Domains of fractions.

Ex: $f(x) = \frac{1}{x}$



Domain is set of all real #s except 0.

$\{x \mid x \neq 0\}$

For fractions, domain is set of all real #s except those that make denominator equal to zero.

If $y = \dfrac{1}{\square}$

Domain $\square \neq 0$ then solve.

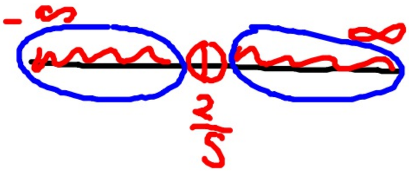
Ex: Find the domain

① $g(x) = \dfrac{x+3}{2-5x}$

$2 - 5x \neq 0$

$-5x \neq -2$

$\boxed{x \neq 2/5}$



$(-\infty, \frac{2}{5}) \cup (\frac{2}{5}, \infty)$

② $f(g) = \dfrac{g+3}{g^2-4}$

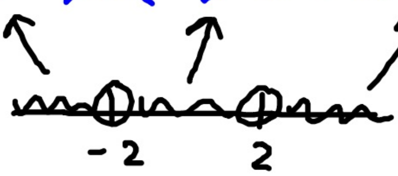
$g^2 - 4 = 0$

$g^2 = 4$

$g = \pm 2$

D: $g \neq \pm 2$

D: $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$



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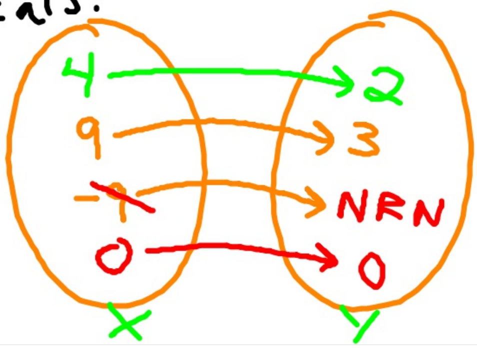
② Domain of radicals.

Ex: $y = \sqrt{x}$



For radicals (even indices), the domain is the set of all $\boxed{\text{non-negative}}$ real numbers.

≥ 0

$$y = \sqrt{\boxed{}}$$

$$\boxed{} \geq 0$$

For $y = \sqrt{x}$

$\boxed{D:\ x \geq 0}$ or $\boxed{D:\ [0, \infty)}$

Ex: Find the domain

① $f(x) = \sqrt{3-5x} \quad +8$

$3-5x \geq 0$

$-5x \geq -3$

D: $\boxed{x \leq 3/5}$

② $g(x) = \sqrt{x^2+3}$

$x^2+3 \geq 0$

$x^2 \geq -3$

D: $\mathbb{R}$

D: $(-\infty, \infty)$

③ radical fraction.

$$y = \frac{1}{\sqrt{\square}} \qquad \square > 0$$

Ex: $y = \dfrac{x}{\sqrt{x+3}}$

D: $x+3>0$

$\boxed{x>-3}$

④ Form sum, difference, product, and quotient of functions.

① Sum: $(f+g)(x) = f(x) + g(x)$

② Difference: $(f-g)(x) = f(x) - g(x)$

③ Product: $(f \cdot g)(x) = f(x) \cdot g(x)$

④ Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

Ex: Given:

$f(x) = x^2 + 3x$, $g(x) = -x + 5$, $h(x) = \sqrt{x+2}$

① Find $(f-g)(x)$

$(f-g)(x) = f(x) - g(x)$

$= (x^2 + 3x) - (-x + 5)$

$= x^2 + 3x + x - 5$

$\boxed{(f-g)(x) = x^2 + 4x - 5}$

② Find $\left(\frac{f}{g}\right)(x)$ and state the domain of $\left(\frac{f}{g}\right)(x)$.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+3x}{-x+5}$$

$$\boxed{\left(\frac{f}{g}\right)(x) = \frac{x^2+3x}{-x+5}}$$

Domain: $x \neq 5$

③ $\left(\frac{h}{g}\right)(x)$ and domain.

$$\left(\frac{h}{g}\right)(x) = \frac{h(x)}{g(x)}$$

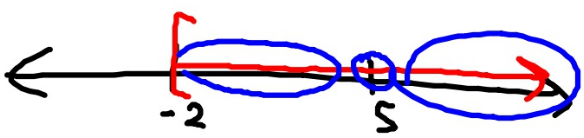
$$\boxed{\left(\frac{h}{g}\right)(x) = \frac{\sqrt{x+2}}{-x+5}}$$

D: $x \neq 5$, $x \geq -2$

$[-2,5) \cup (5,\infty)$

$x+2 \geq 0$

$x \geq -2$



④ $(fg)(-2)$

$$(fg)(-2) = f(-2)\,g(-2)$$

$$= \left((-2)^2 + 3(-2)\right)\left(-(-2) + 5\right)$$

$$= (-2)(7)$$

$$\boxed{(fg)(-2) = -14}$$