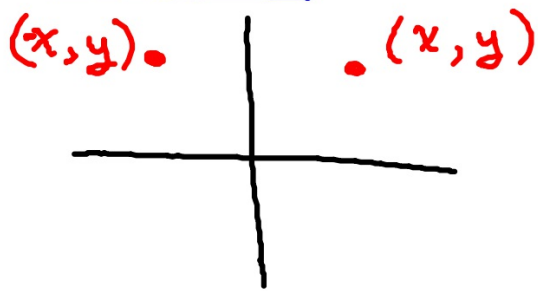


3.3 Properties of Functions.

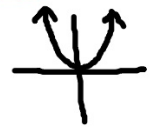
I. Determine even and odd functions from a graph

① A function f is **even** if, for every x in the domain, the number $-x$ is also in the domain and $f(-x) = f(x)$.

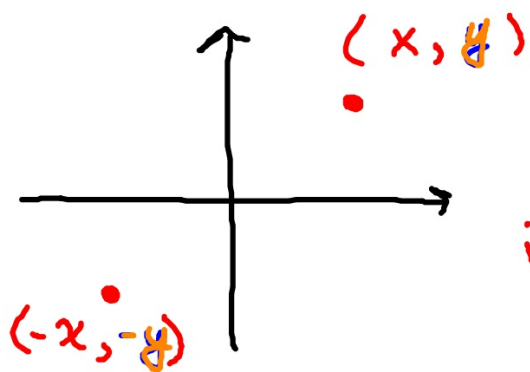


even functions are sym. w.r.t y -axis

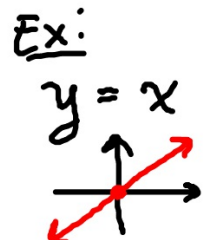
$$y = x^2$$



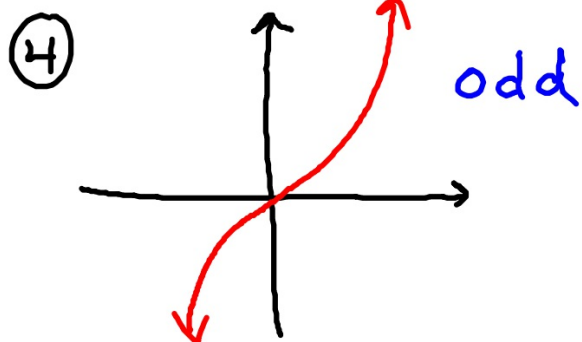
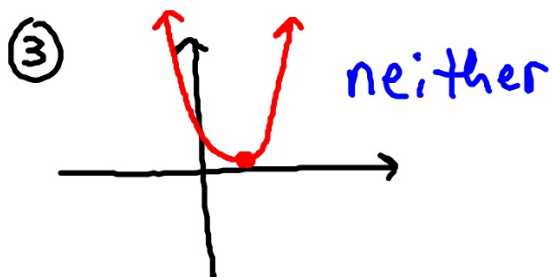
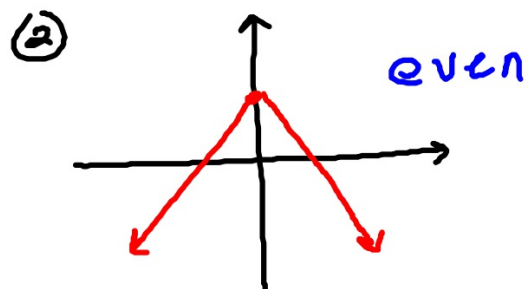
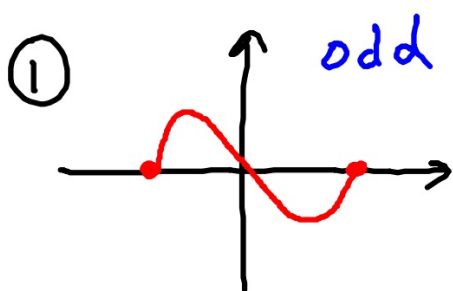
② A function f is odd if, for every number x in the domain, the number $-x$ is also in the domain and $f(-x) = -f(x)$.



Any odd function is sym. w.r.t the origin



Ex: Determine whether the function is even, odd, or neither.



II. Determine even and odd functions from an equation.

① $f(-x) = f(x)$ even

② $f(-x) = -f(x)$ odd.

otherwise, neither

Ex: Determine whether the function is even, odd, or neither.

① $f(x) = 3x^4 - 5x^2 + 7$

$$f(-x) = 3(-x)^4 - 5(-x)^2 + 7$$

$$f(-x) = 3x^4 - 5x^2 + 7 = f(x) \text{ (even.)}$$

$$\textcircled{2} \quad f(x) = 4x^3 - 2x$$

$$f(-x) = 4(-x)^3 - 2(-x)$$

$$f(-x) = -4x^3 + 2x$$

← not even

$$f(-x) = -(4x^3 - 2x)$$

$$f(-x) = - \underbrace{(4x^3 - 2x)}_{f(x)}$$

odd function.

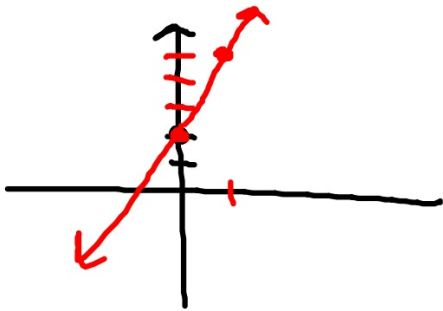
$$\textcircled{3} \quad f(x) = \underline{\underline{3x}} + \underline{\underline{2}}$$

$$f(-x) = 3(-x) + 2$$

$$f(-x) = -3x + 2 \quad \leftarrow \text{not even}$$

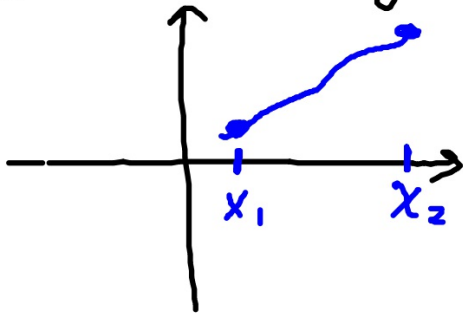
$$f(-x) = - (3x - 2) \quad \leftarrow \text{not odd.}$$

\therefore neither.



III. Increasing, decreasing, and constant.

① Increasing



The y -values are increasing from x_1 to x_2 .

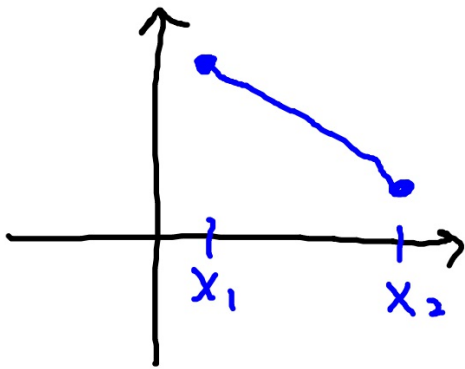
function is increasing on interval

(x_1, x_2)

Note: always use parentheses.

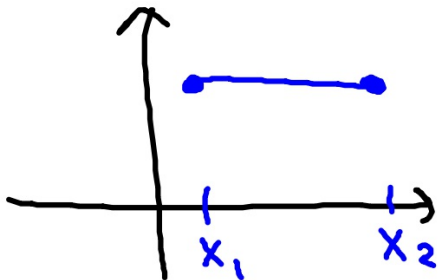
② Decreasing

Read from left to right



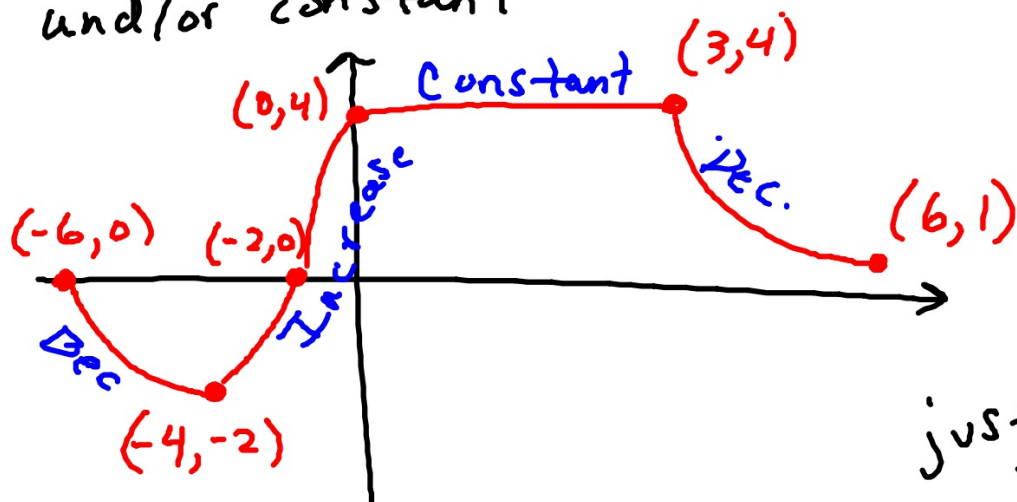
Function is decreasing
 (x_1, x_2)

③ Constant



function is constant
 (x_1, x_2)

Ex: Determine the intervals where the graph is increasing, decreasing, and/or constant



just use x
values

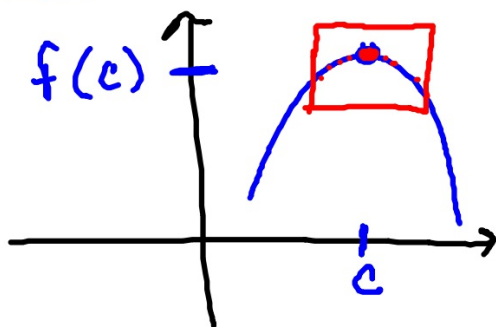
Increasing: $(-4, 0)$

Decreasing: $(-6, -4) \cup (3, 6)$

Constant: $(0, 3)$

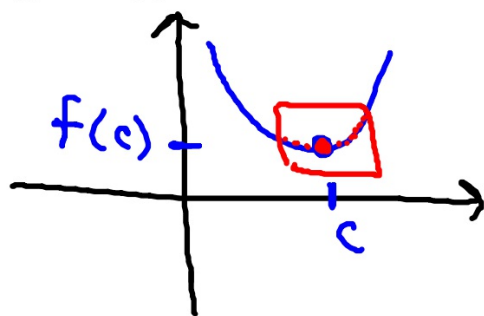
IV. Local maxima and local minima

① Local maximum



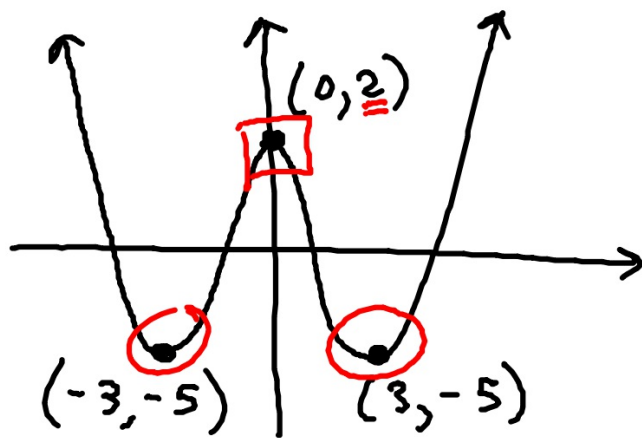
local maximum
value is $f(c)$
and is located at
 $x=c$

② Local minimum



local min value is $f(c)$
located at $x=c$.

Ex: list the local max and min

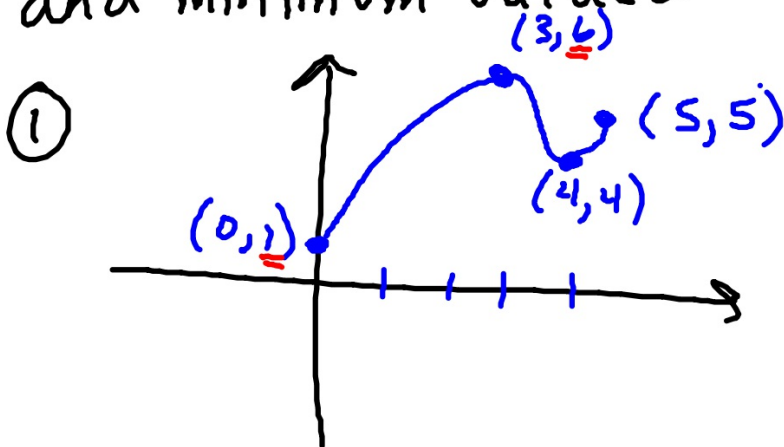


local max value: 2
local min value: -5

V. Absolute maximum and minimum

Largest or smallest y -value in the entire domain of the function.

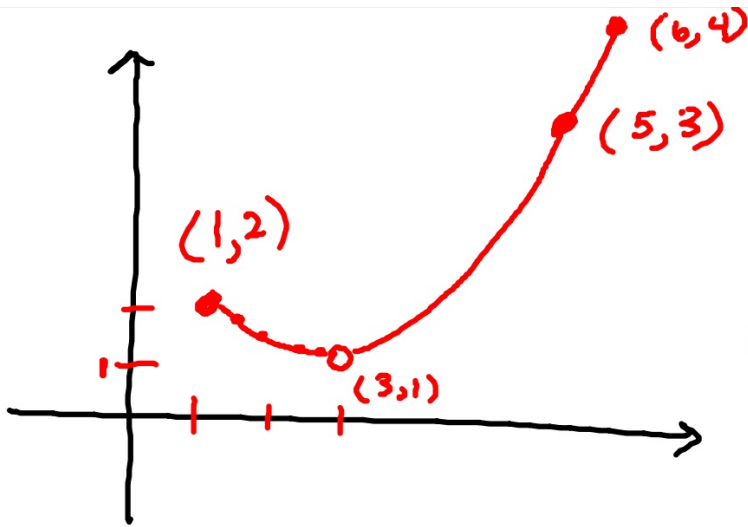
Ex: Determine the absolute maximum and minimum values.



Abs. max: 6

Abs. min: 1

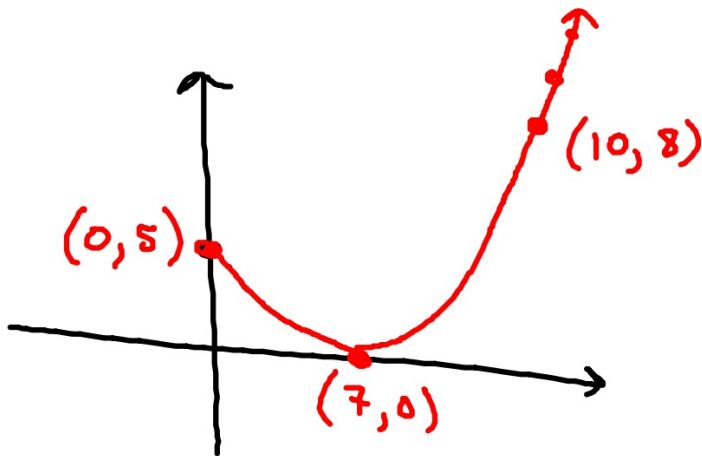
②



Abs. max: 4

Abs. min: none

③

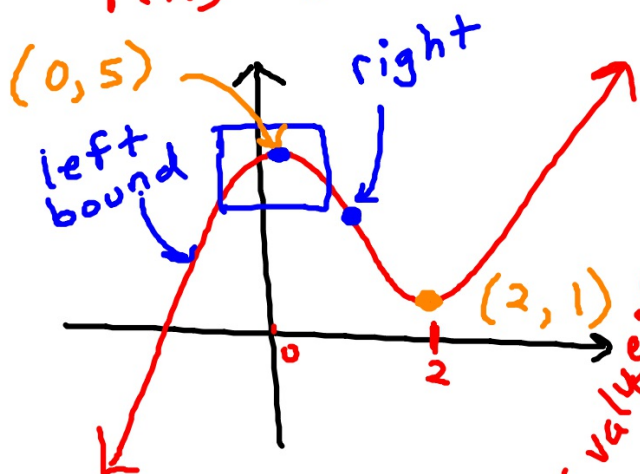


Abs. max: none

Abs min: 0

VI. Graphing calculator for finding extrema.

① $f(x) = x^3 - 3x^2 + 5$



Abs. max none
Abs. min none
local max 5
local min 1

Inc: $(-\infty, 0) \cup (2, \infty)$
Dec: $(0, 2)$
Constant: none

② $g(x) = 6x^3 - 12x + 5$

Find: (Round to 3 decimal places).

Abs max none

Abs min none

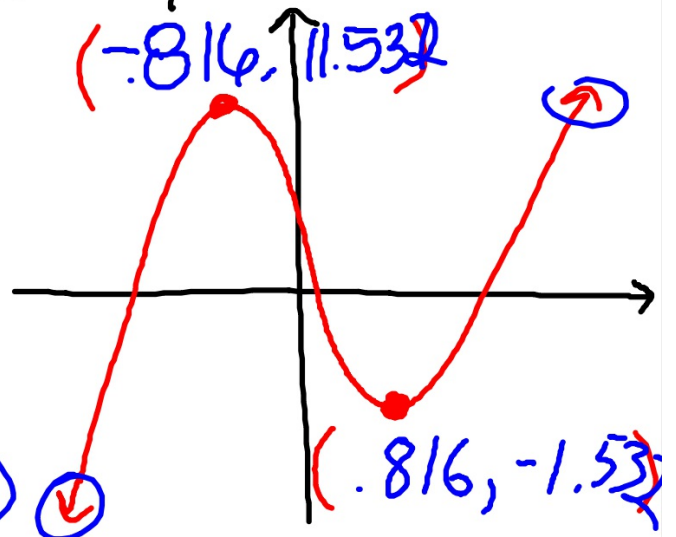
local max 11.532

local min -1.532

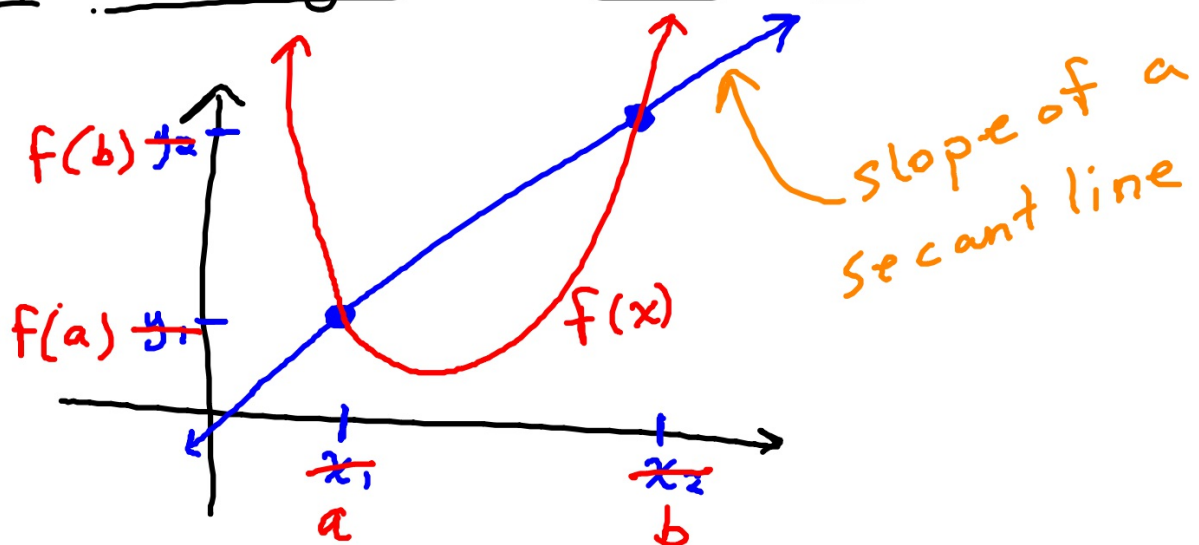
Inc: $(-\infty, -0.816), (0.816, \infty)$

Dec: $(-0.816, 0.816)$

Constant: none



VII. Average rate of change \rightarrow slope



Average rate of change

$$A.R.C = \frac{f(b) - f(a)}{b - a}$$

Ex: Find the average rate of change

for the function $f(x) = -3x^2 + x - 5$

from -1 to 2 .
 a b

A.R.C

$$\frac{f(2) - f(-1)}{2 - (-1)}$$

$$= \frac{-15 - (-9)}{2 - (-1)}$$

$$= \frac{-15 + 9}{2 + 1} = \frac{-6}{3} = \boxed{-2}$$

$$f(2) = -3(2)^2 + (2) - 5 = -15$$

$$f(-1) = -3(-1)^2 + (-1) - 5 = -9$$

$$(-1, -9)$$

$$(2, -15)$$