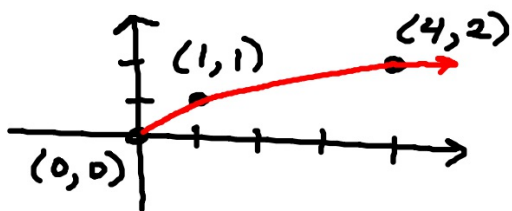


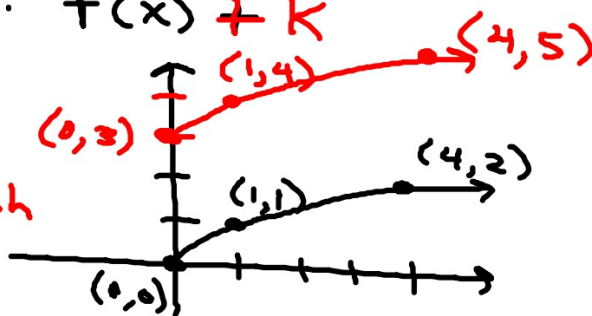
3.5 Graphing techniques: Transformations

Remember: $f(x) = \sqrt{x}$

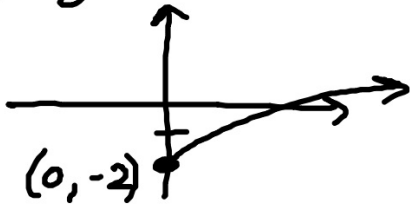


I. Vertical shift: $f(x) + k$

Ex: $y = \sqrt{x} + 3$
 $y_1 = \sqrt{x}$
 $y_2 = \sqrt{x} + 3$
shifts graph
up 3.



Ex: $y = \sqrt{x} - 2$

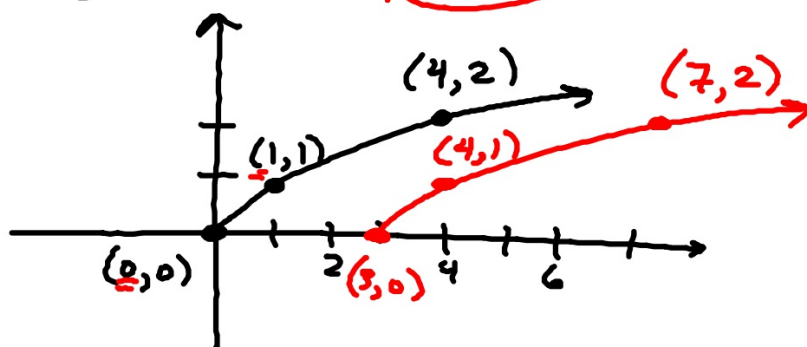


Summary:

- ① $f(x) + k$ shifts $f(x)$ up k units
- ② $f(x) - k$ shifts $f(x)$ down k units.

II. Horizontal shift: $f(x-h)$

Ex: $y = \sqrt{x-3}$



Note: ADD 3 to all x-values.

$$y = \sqrt{x - (-3)}$$

$$y = \sqrt{x+3}$$

← shift 3 units left

summary: $f(x-h)$

① $f(x-h)$ shifts $f(x)$ right h units

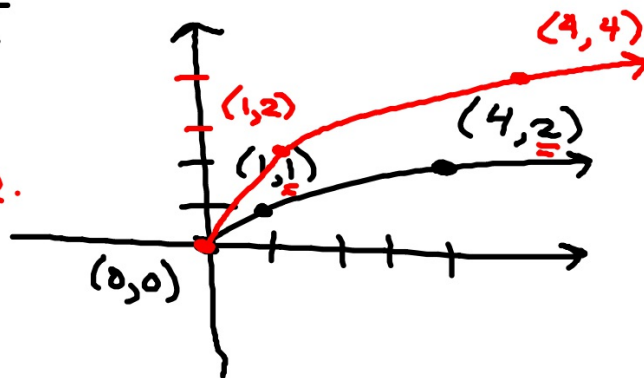
② $f(x+h)$ shifts $f(x)$ left h units.

III. Vertical stretch/compression: a $\cdot f(x)$

Ex: $y = 2\sqrt{x}$

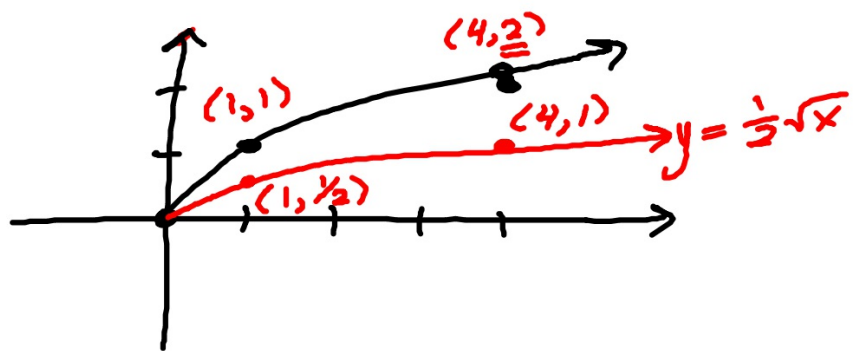
Stretch by
a factor of 2.

~~*~~ multiply all
y's by 2.



Ex: $y = \frac{1}{2}\sqrt{x}$

vertical
compression



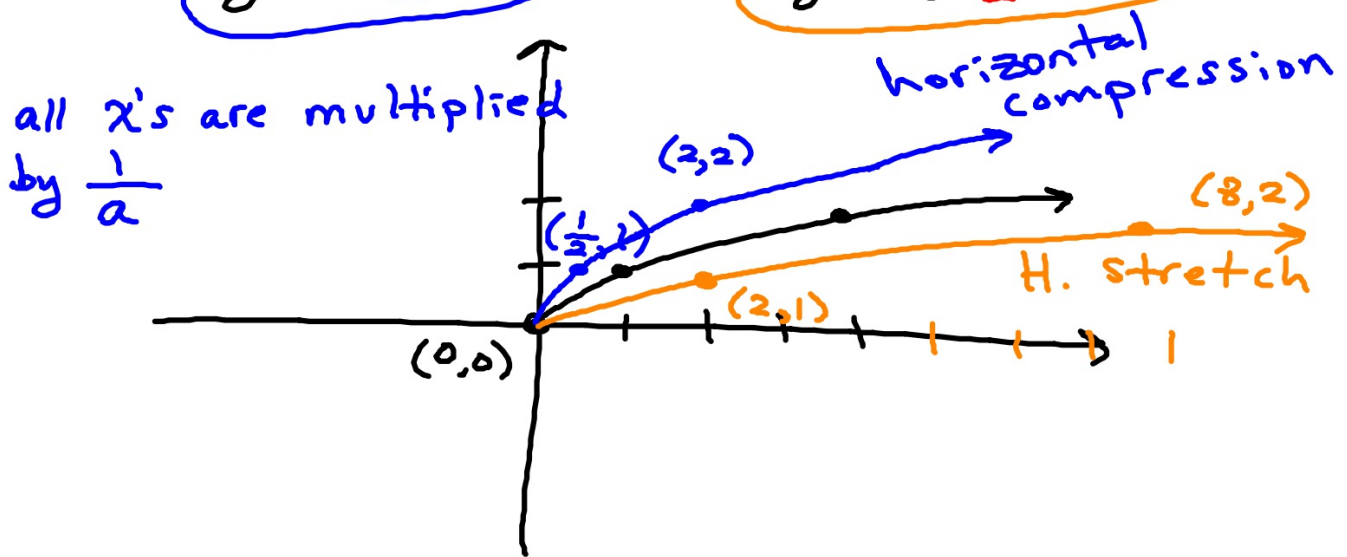
Summary: $a \cdot f(x)$

① Vertical stretch if $|a| > 1$

② Vertical compression if $|a| < 1$

IV. Horizontal stretch/compression $f(ax)$

Ex: $y = \sqrt{2x}$ vs $y = \sqrt{\frac{1}{2}x}$



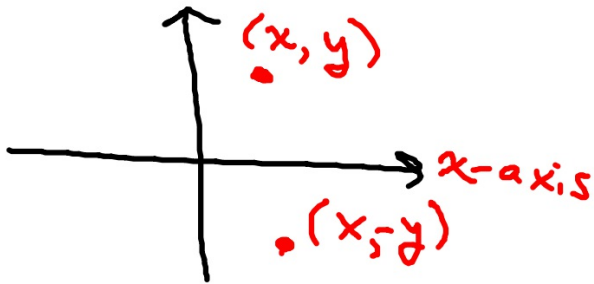
Summary: $f(a \cdot x)$

① Horizontal Stretch if $|a| < 1$

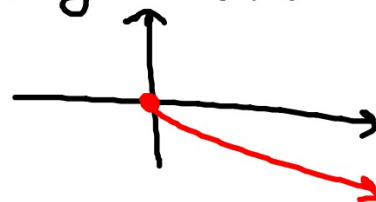
② Horizontal Compression if $|a| > 1$.

V. Reflections.

① Reflection across x -axis: $-f(x)$

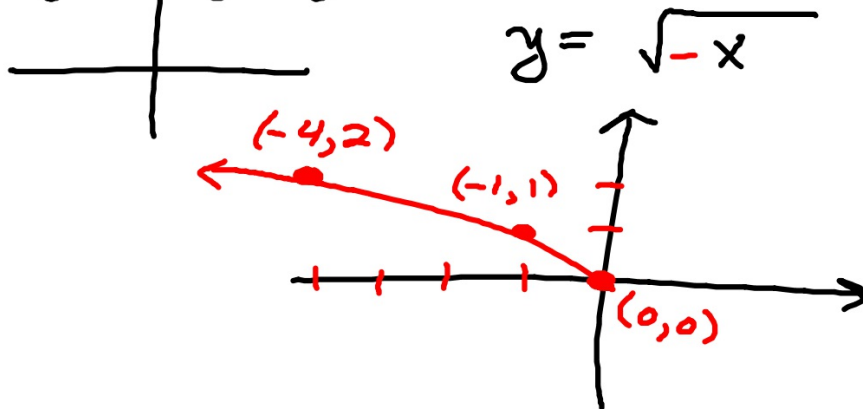


Ex: $y = -\sqrt{x}$



② Reflect across y-axis: $f(-x)$

$(-x, y)$ (x, y)



General Summary

$$y = \pm a \cdot f(\pm b(x-h)) \pm k$$

vertical
Stretch/
Compression

reflect
across
x-axis

horizontal
Shift

$x-h \rightarrow$ right

$x+h \rightarrow$ left

horizontal
Stretch/
Compression

reflects across
y-axis

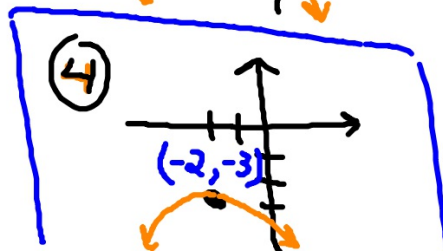
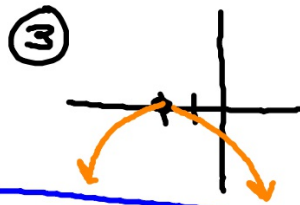
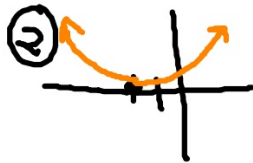
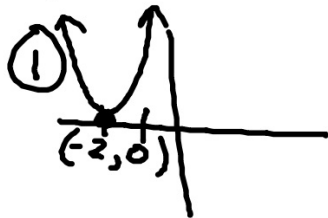
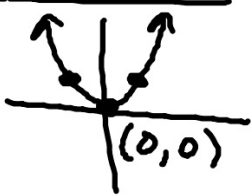
vertical shift
 $+k$ up
 $-k$ down

Ex: Graph using transformations

$$\textcircled{1} \quad f(x) = -\frac{1}{2}(x+2)^2 - 3$$

Annotations for the equation above:
- A blue arrow points from the text "reflect across x-axis" with a circled 3 to the negative sign in front of the fraction.
- An orange arrow points from the text "vertical compression" with a circled 2 to the denominator 2.
- A red arrow points from the text "left 2" with a circled 1 to the +2 inside the parentheses.
- A red arrow points from the text "down 3" with a circled 4 to the constant term -3.

Parent: $y = x^2$



② $g(x) = -2\sqrt{-x+1} + 3$

$g(x) = -2\sqrt{-(x-1)} + 3$

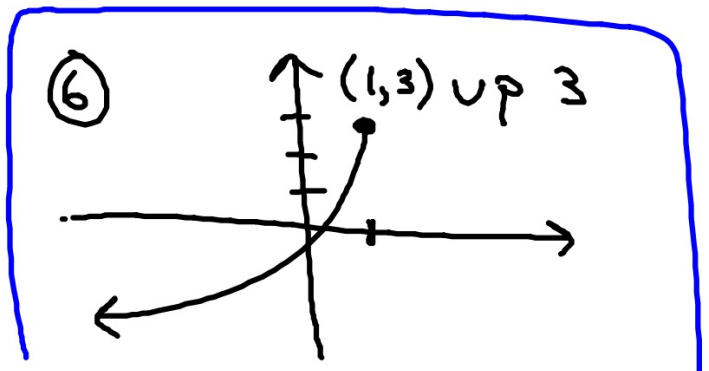
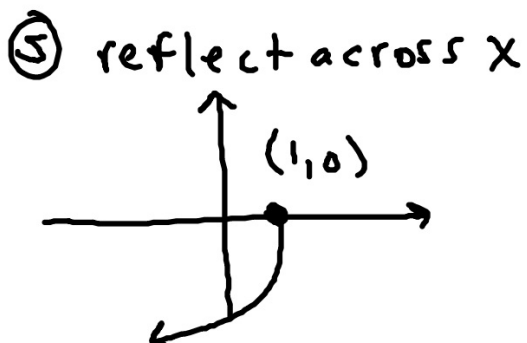
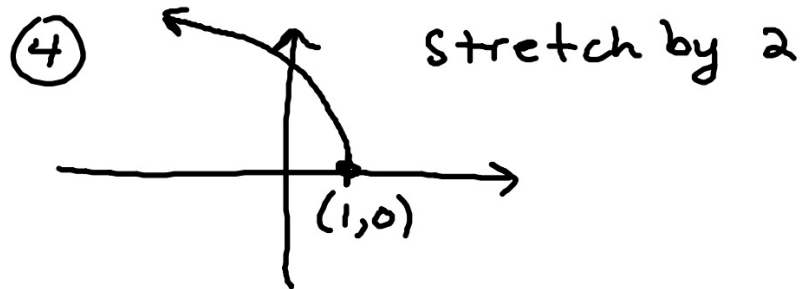
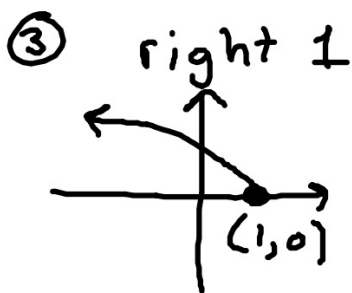
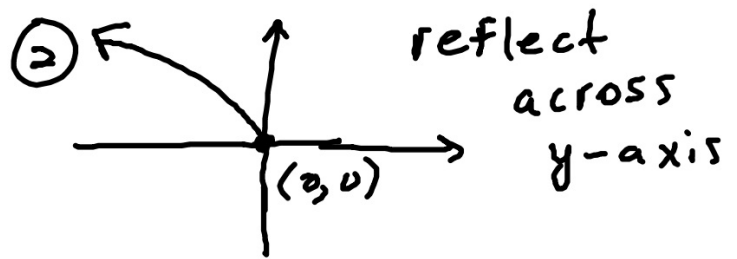
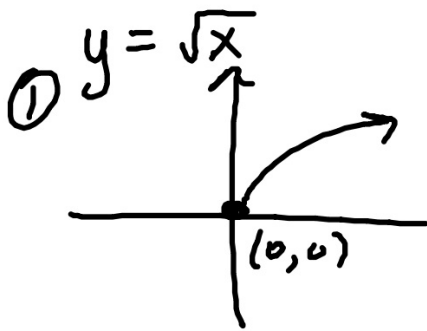
reflect
x-axis

Stretch by 2

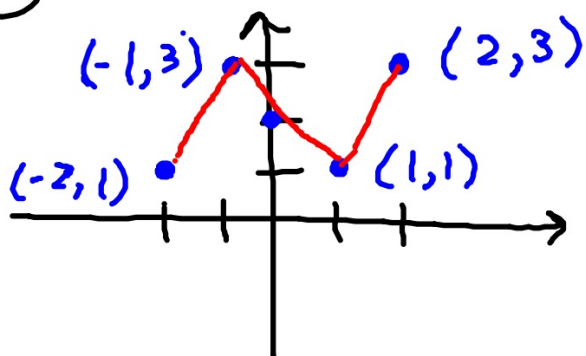
right
1

reflect across
y-axis

up 3



③ Given $f(x)$



graph $-2 \cdot f(x-1) - 3$

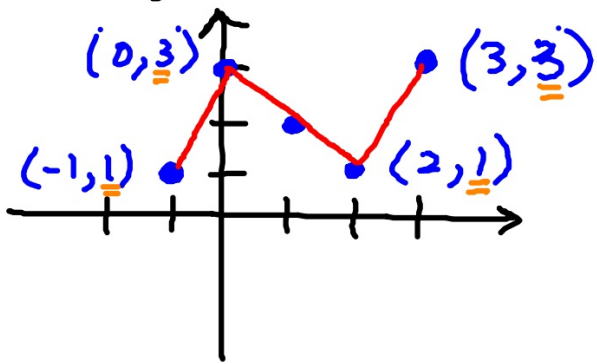
reflect across x-axis

stretch by 2

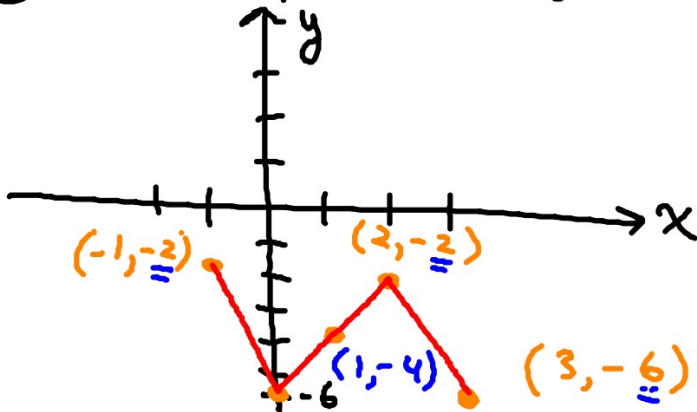
right 1

down 3

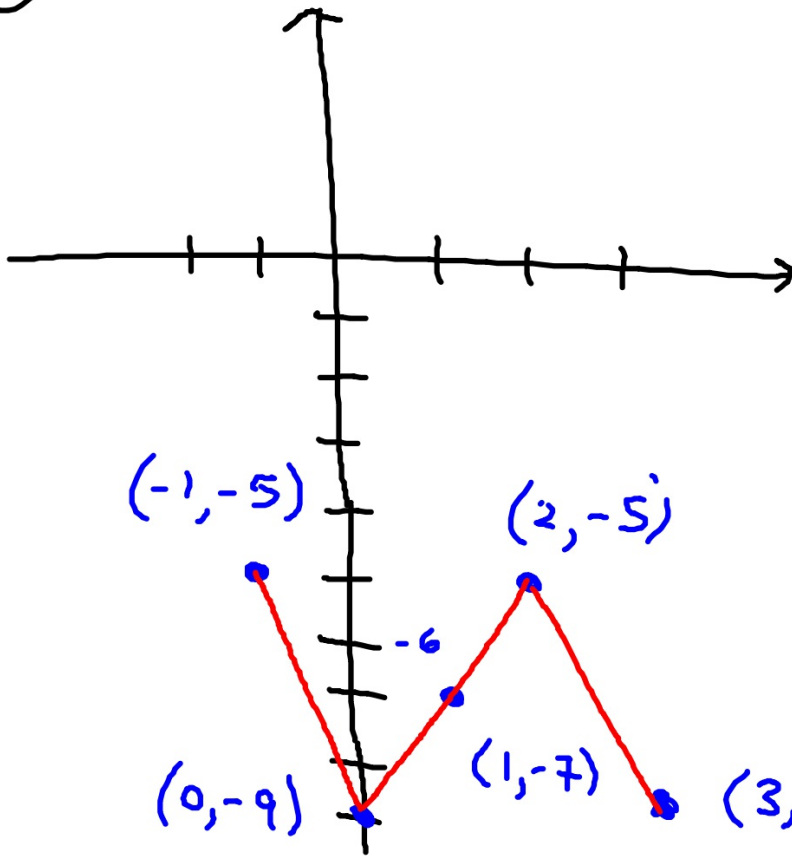
① right 1 (add 1 to all x 's)



② stretch/reflect (multiply y 's by -2)



③ down 3 units (subtract 3 from y 's).



$$D: [-1, 3]$$

$$R: [-9, -5]$$

$$-2 \cdot f(x-1) - 3$$

multiply y's by -2

Add 1 to x's

subtract 3 from y's

$$(-2, 1) \rightarrow (-2+1, 1(-2)-3) \rightarrow (-1, -5)$$

$$(2, 3) \rightarrow (2+1, 3(-2)-3) \rightarrow (3, -9)$$