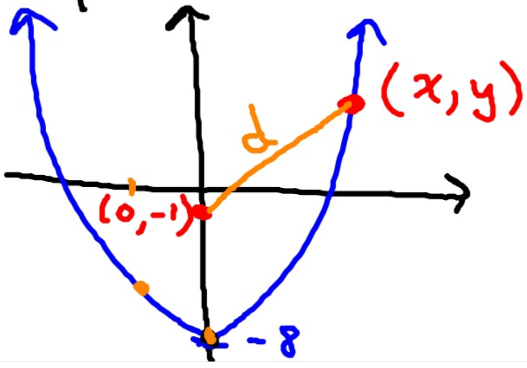
3.6 Mathematical Models: Building functions.

Ex 1 Let $P(x,y)$ be a point on the graph of $y = x^2 - 8$.

a) Express the distance d from P to the point $(0,-1)$ as a function of x.



$$d = \sqrt{(x-0)^2 + (y-(-1))^2}$$

$$d = \sqrt{x^2 + (y+1)^2}$$

$$d = \sqrt{x^2 + (x^2 - 8 + 1)^2}$$

$$d = \sqrt{x^2 + (x^2 - 7)^2}$$

$$d = \sqrt{x^2 + x^4 - 14x^2 + 49}$$

$$d(x) = \sqrt{x^4 - 13x^2 + 49}$$

b) what is d if $x = 0$?

$$d(0) = \sqrt{(0)^4 - 13(0) + 49} = \sqrt{49} = 7$$

c) what is d if $x = -1$?

$$d(-1) = \sqrt{37} \approx 6.08$$

d) Graph $d(x)$. For what values of x is $d(x)$ smallest?

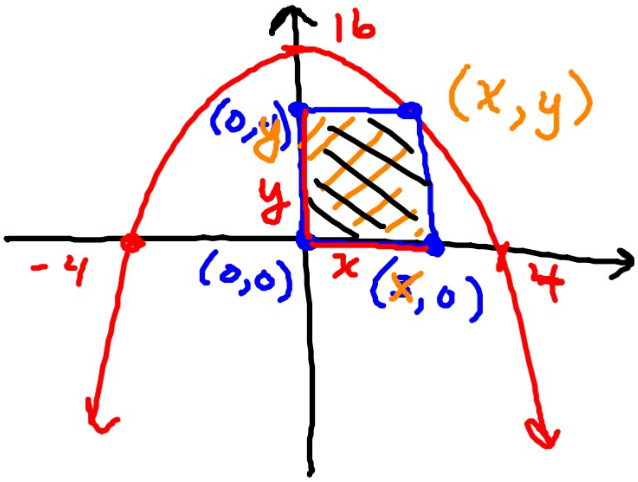
$$Y_1 = \sqrt{x^4 - 13x^2 + 49}$$

$x = -2.55$

$x = 2.55$

#2

A rectangle has one corner in quadrant I on the graph of $y = -x^2+16$, another at the origin, a third on the positive x-axis, and a fourth on the positive y-axis.



a) Express the area A as a function of x.

$A = xy$

$A = x \cdot (-x^2+16)$

$$A(x) = -x^3+16x$$

b) What is the domain of $A(x)$?

$x > 0$ and $x < 4$

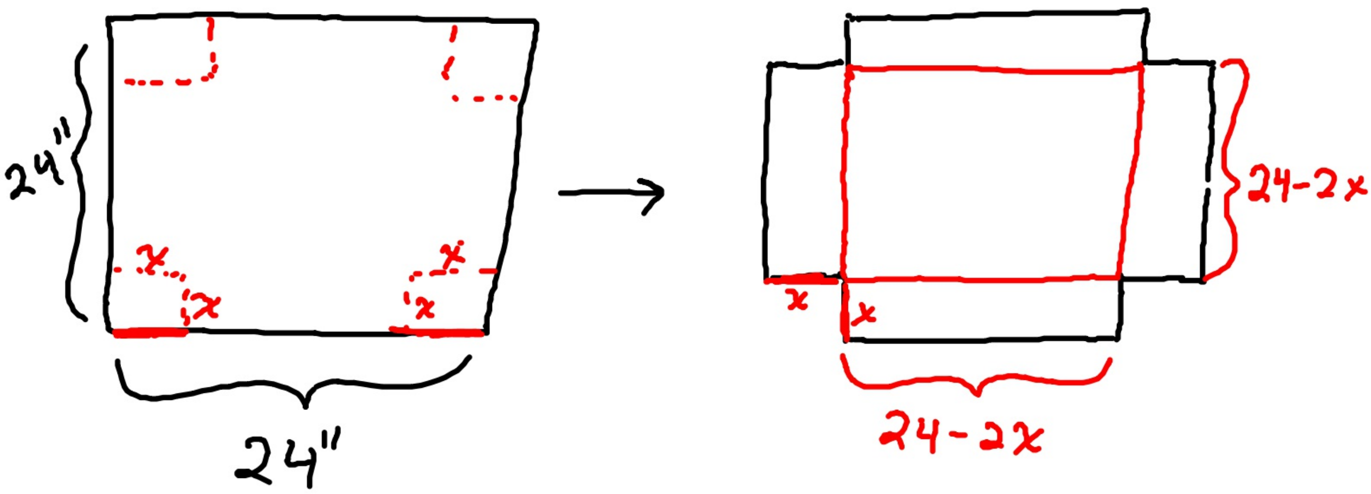
$0 < x < 4$ or $(0, 4)$

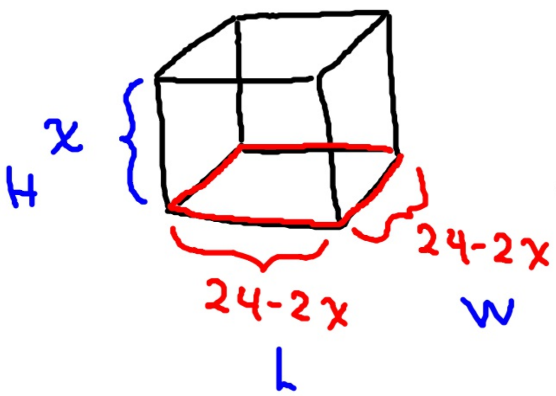
c) Graph $A(x)$. For what positive value of x is A largest?

$$\boxed{x \approx 2.31}$$

Ex 3 An open box with a square base is to be made from a square piece of cardboard 24 inches on a side by cutting out a square from each corner and turning up the sides.

a) Express the volume of the box as a function of the length x of the side of the square cut from each corner.





$V = L \cdot W \cdot H$

$V = (24-2x)(24-2x)(x)$

(a) $V(x) = x(24-2x)^2$

(b) What is the volume if a <u>3-inch square</u> is cut out?

$x=3$, $V(3)=3(24-2\cdot 3)^2 \Rightarrow$ $\boxed{972 \text{ in}^3}$

(c) What is the volume if a 10-inch square is cut out?

$\boxed{V(10)=160 \text{ in}^3}$

d) What is the domain of V?

$x>0$ and $x<12$

$\boxed{0<x<12}$

$24-2x=0$

$x=12$

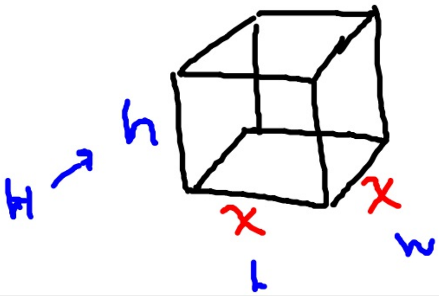
e) Graph $V(x)$. For what value of x is V largest?

$x \approx 4$ in

$y = \underline{\underline{1024}}$

Ex4 An open box with a square base is required to have a volume of 10 cubic feet.

(a) Express the amount A of material used to make such a box as a function of the length x of a side of the square base.



$A = x^2 + 2xh + 2xh$

$A = x^2 + 4xh$

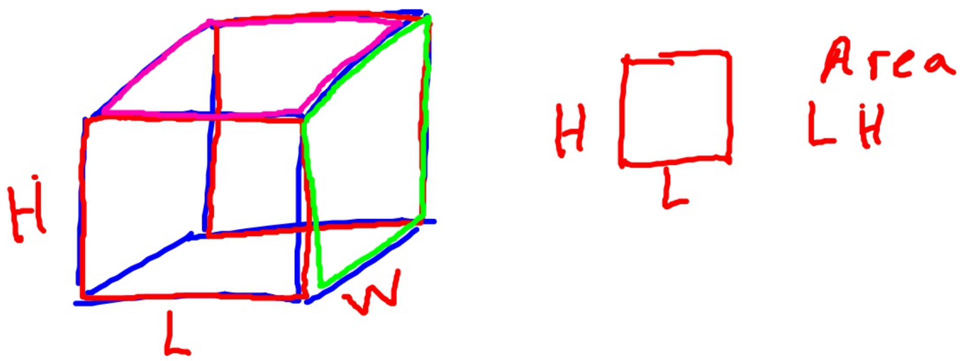
$A = x^2 + x(10/x^2)$

$V = LWH$

$10 = x \cdot x \cdot h$

$10 = x^2 h$

$h = 10/x^2$



SA: front/back + left/right + top/bottom

SA = 2LH + 2WH + 2LW

$$A(x) = x^2 + \frac{40}{x}$$

surface area (A) — length of square base (x).

(b) How much material is required for a base of 1 foot by 1 foot?

$x = 1 \quad A(1) = (1)^2 + \frac{40}{(1)} = 41 \text{ ft}^2$

(c) Graph $A(x)$. For what value of x is A smallest?

$x \approx 2.71 \text{ ft}$