Apr. 10, 2014

Sect. 4-3 & 4

Trig Identities

Sum/Diff IDs

Double/Half Angle IDs

Prod-to-Sum

Sum-to-Prod IDs

$$\sin(x+y) = \sin x\cos y + \sin y\cos x$$

$$\sin(x-y) = \sin x\cos y - \sin y\cos x$$

$$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$

$$\cos(x-y) = \cos x\cos y + \sin x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Find the exact value of

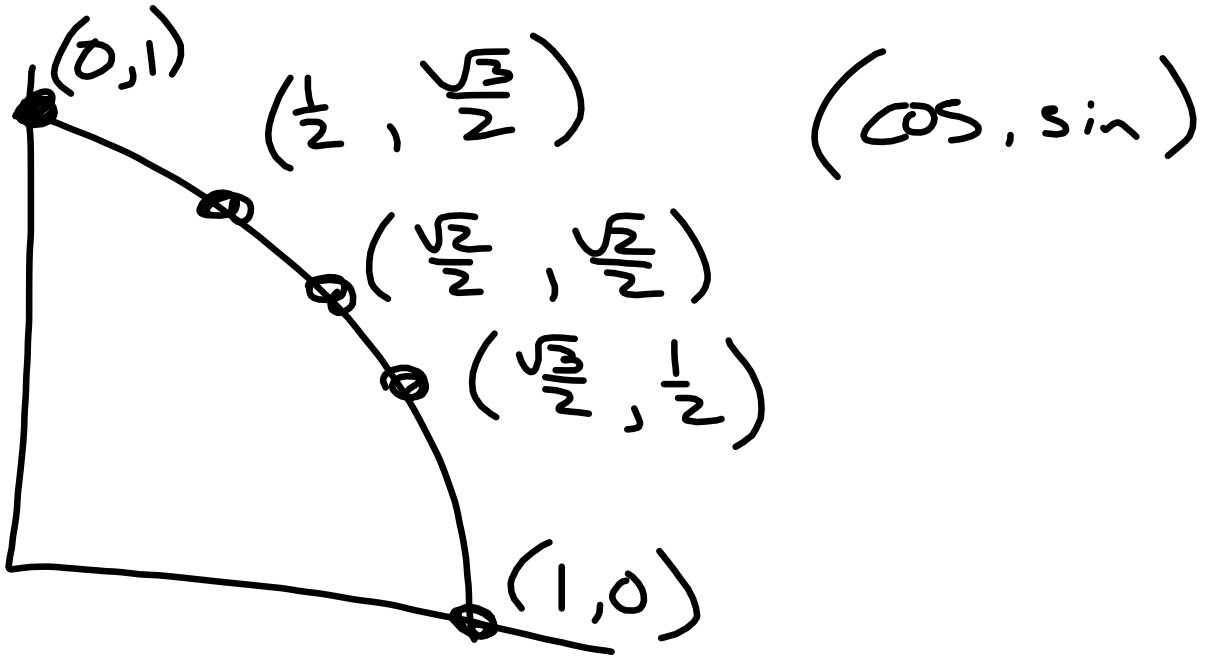
$\sin 15^\circ =$

$\sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$


(0,1)
$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
(1,0)
(cos, sin)

$$\cos 75^\circ = \cos(45^\circ + 30^\circ)$$

$$\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos 105^\circ$$

$$\cos(60^\circ + 45^\circ)$$

$$\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\cos 60^\circ =$$

$$\cos(30^\circ + 30^\circ)$$

$$\cos 30^\circ \cos 30^\circ - \sin 30^\circ \sin 30^\circ$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2}$$

$$\frac{3}{4} - \frac{1}{4}$$

$$\frac{2}{4}$$

$$\frac{1}{2}$$

$$\cos 2x$$

$$\cos(x+x)$$

$$\cos x \cos x - \sin x \sin x$$

$$\cos^2 x - \sin^2 x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

Double-Angle Form

$$\cos 2x = \cos^2 x - \sin^2 x \quad (1)$$

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x = 1 - \cos^2 x$$
$$\cos^2 x = 1 - \sin^2 x$$

$$= 1 - \sin^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x \quad (2)$$
$$= 2\cos^2 x - 1 \quad (3)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Find $\cos 2x$ and $\tan 2x$

if $\sin x = \frac{4}{5}$ and $\frac{\pi}{2} < x < \pi$

Q2

$\cos 2x = 1 - 2\sin^2 x$

$= 1 - 2\left(\sin x\right)^2$

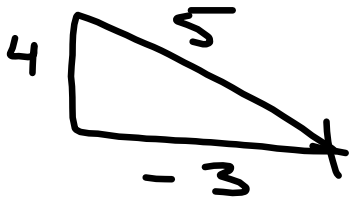
$= 1 - 2\left(\frac{4}{5}\right)^2$

$= 1 - 2\left(\frac{16}{25}\right)$

$= 1 - \frac{32}{25} \Rightarrow = -\frac{7}{25}$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$\sin x = \frac{4}{5}$ Q2



$$\tan x = -\frac{4}{3}$$

$$\tan 2x = \frac{2\left(-\frac{4}{3}\right)}{1-\left(-\frac{4}{3}\right)^2} = \frac{-\frac{8}{3}}{1-\frac{16}{9}}$$

$$= \frac{+\frac{8}{3}}{+\frac{7}{9}} = \frac{8}{3}\cdot\frac{9}{7}^{3} = \frac{24}{7}$$

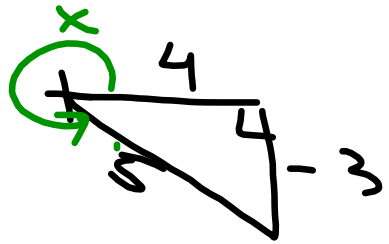
Find $\sin 2x$ if $\tan x = -\frac{3}{4}$

$-\frac{\pi}{2} < x < 0$

Q4

$\sin 2x = 2 \sin x \cos x$

$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$



$= -\frac{24}{25}$

Find $\cos 2x$ same info

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= \left(\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{7}{25}$$

Half-Angle Formulas

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$\pm$ You pick the sign for the Q.

Find the exact value of

$\sin 15^\circ$ Q1

$$\sin\frac{30^\circ}{2} = +\sqrt{\frac{1-\cos 30^\circ}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$