Mar. 1, 2016

Sects. 6-4

Vectors

$R: (1, 2)$

$S: (4, 4)$

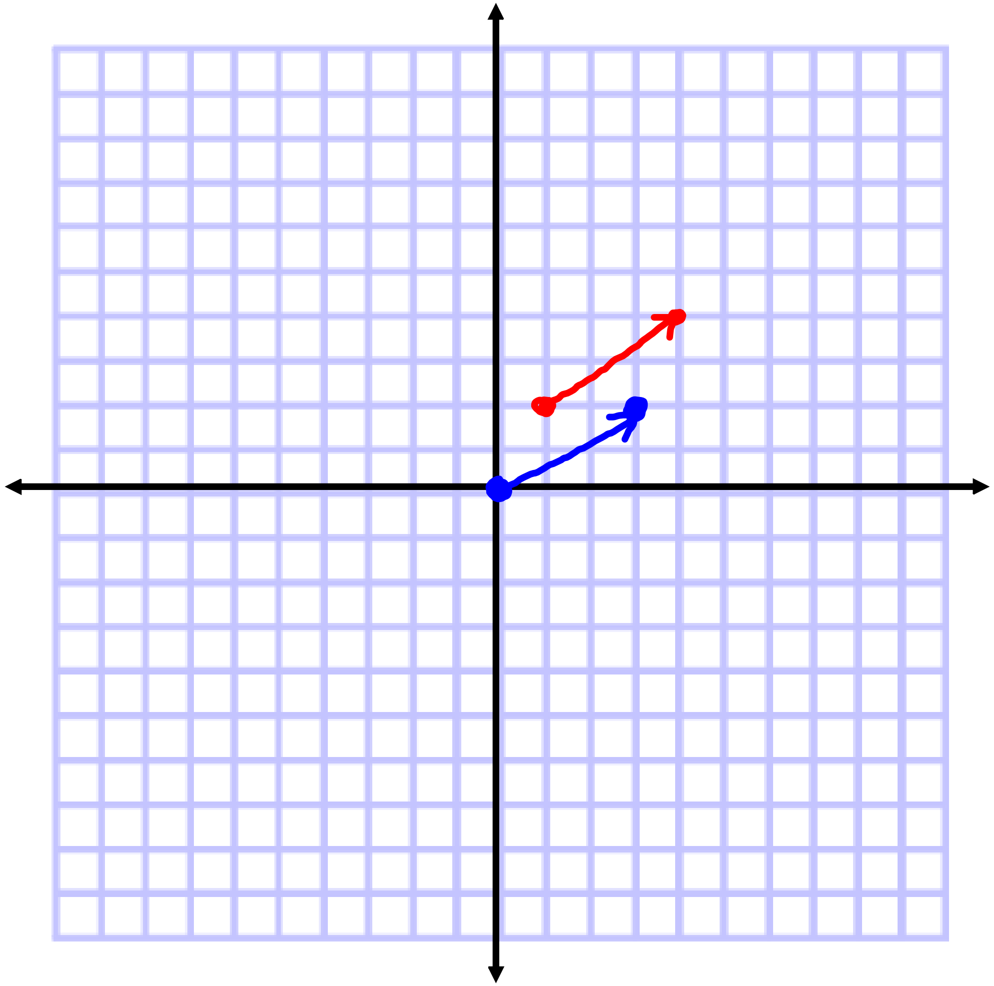
$P: (0, 0)$

$Q (3, 2)$

$\overrightarrow{PQ}$

$\overrightarrow{RS}$

$\overrightarrow{RS} = \overrightarrow{PQ}$



We can write vectors using $\langle x, y \rangle$

$\overrightarrow{PQ} = \langle 3, 2 \rangle$

$\overrightarrow{RS} = \langle 3, 2 \rangle$

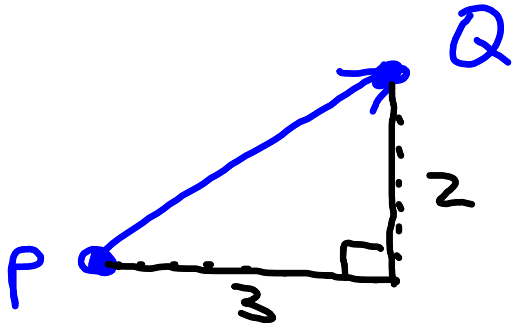
$\overrightarrow{PQ} = \overrightarrow{RS}$

The length of a vector
is called the
magnitude.

Magnitude

notation: $\| \quad \|$

$\|\overrightarrow{PQ}\| =$



$(m\overline{PQ})^2 = 3^2 + 2^2$

$(m\overline{PQ})^2 = 9 + 4 = 13$

$m\overline{PQ} = \sqrt{13}$

$\|\overrightarrow{PQ}\| = \sqrt{13}$

In General

$$\| \langle x, y \rangle \|$$

$$= \sqrt{x^2 + y^2}$$

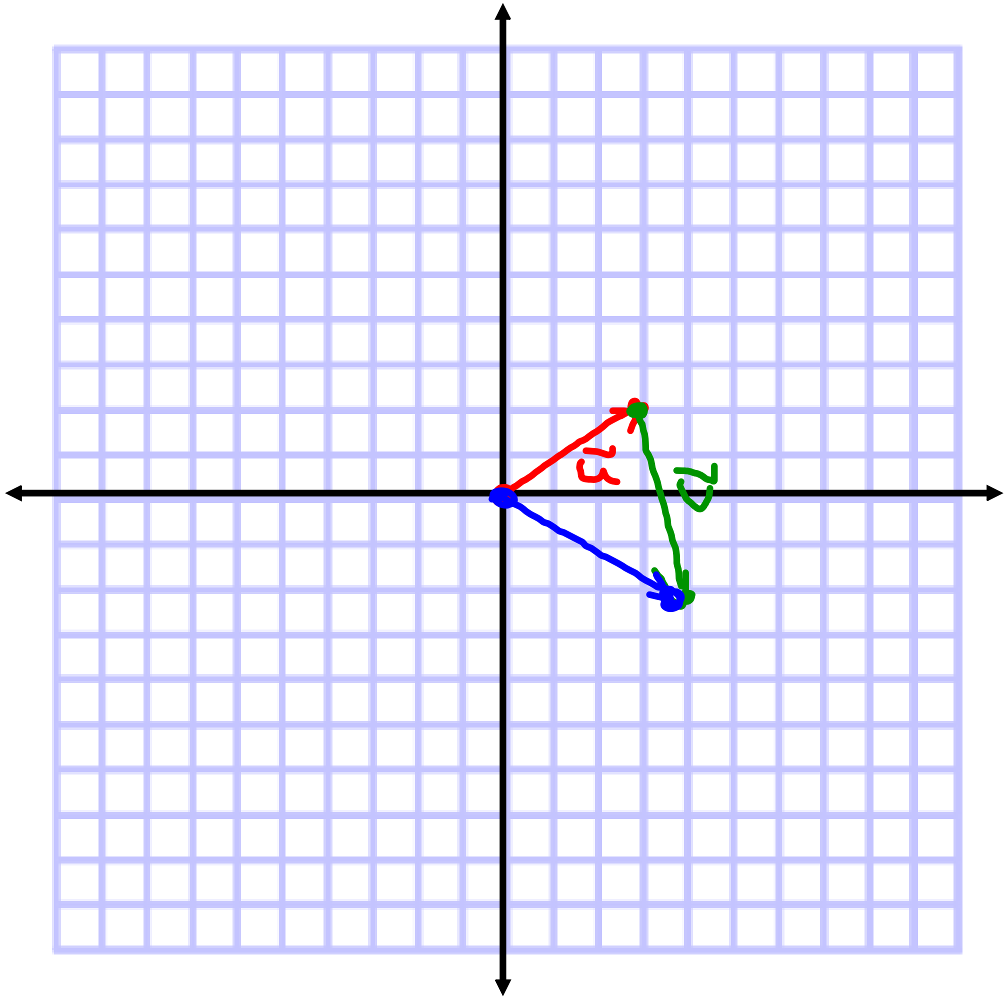
Vector Addition

$\vec{u} = \langle 3, 2 \rangle$

$\vec{v} = \langle 1, -4 \rangle$

$\vec{u} + \vec{v} =$

$\langle 4, -2 \rangle$



In General
Add / Subt.

$\vec{u} = \langle x_1 , y_1 \rangle$

$\vec{v} = \langle x_2 , y_2 \rangle$

$\vec{u} \pm \vec{v} = \langle x_1 \pm x_2 , y_1 \pm y_2 \rangle$

Scalar Mult.

$\vec{w} = \langle 1, -4 \rangle$

Find $3\vec{w} = 3\langle 1, -4 \rangle$

$= \langle 3, -12 \rangle$

Unit Vectors

Defn: Vector with a mag. of 1.

Given $\vec{v}$. The unit vector in the same direction as $\vec{v}$ is

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$$

Give $\vec{v} = \langle 3, 4 \rangle$

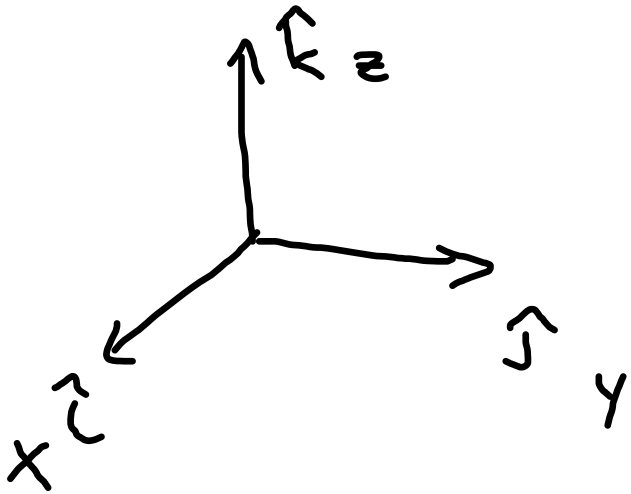
Find its unit vector.

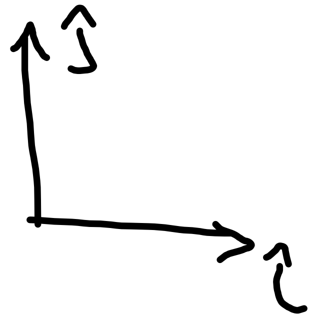
$$\|\vec{v}\| = \sqrt{3^2 + 4^2} = 5$$

$$\vec{u} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

Special Vectors

$\hat{i}$ $\hat{j}$ $\hat{k}$





So

$\vec{p} = \langle 3, 4 \rangle$ can be written

$3\hat{i} + 4\hat{j}$

$\vec{q} = \langle 5, -2 \rangle$

$5\hat{i} - 2\hat{j}$

$$\vec{p} + \vec{q} = 3\hat{i} + 4\hat{j}$$

$$+\ 5\hat{i} - 2\hat{j}$$

$$= 8\hat{i} + 2\hat{j}$$

$$\vec{p} - \vec{q} = 3\hat{i} + 4\hat{j}$$

$$-\ (5\hat{i} - 2\hat{j})$$

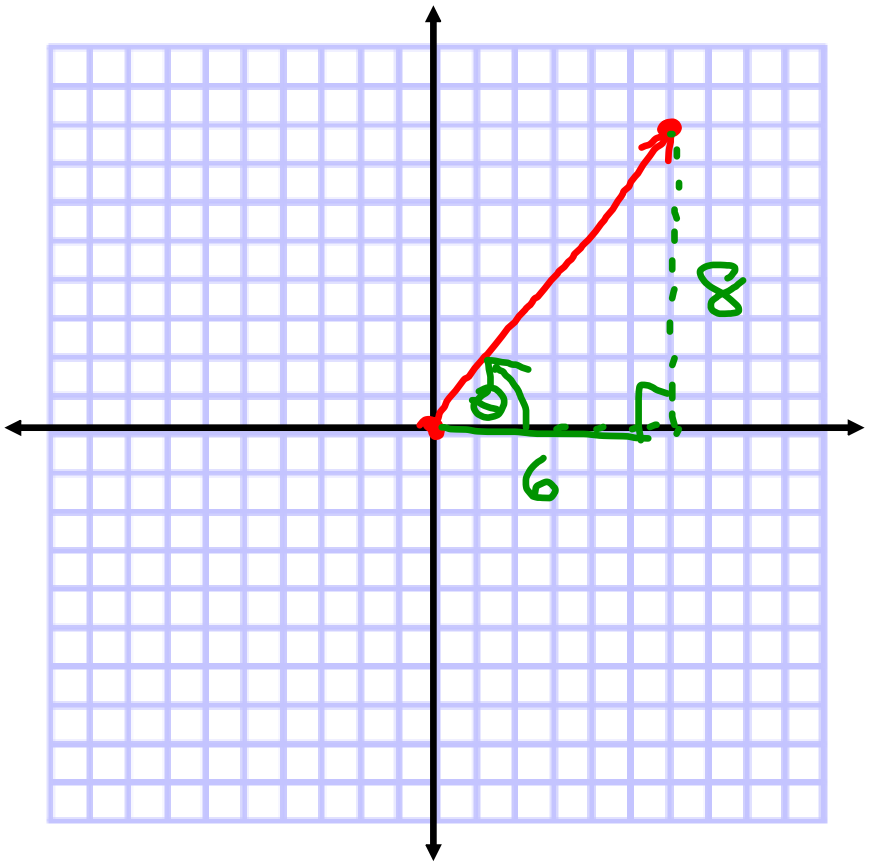
$$-2\hat{i} + 6\hat{j}$$

How do you find the direction?

$\vec{m} = 6\hat{i} + 8\hat{j}$

$\tan\theta = \frac{8}{6}$

$m\angle\theta = 53.1^\circ$

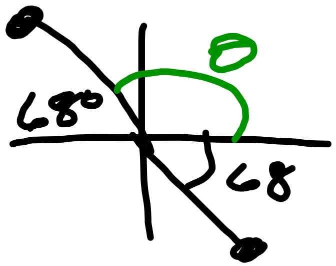


In general, to find the angle (in st. position) of a vector:

$$m\angle\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Find θ for

$-2\hat{i} + 5\hat{j}$



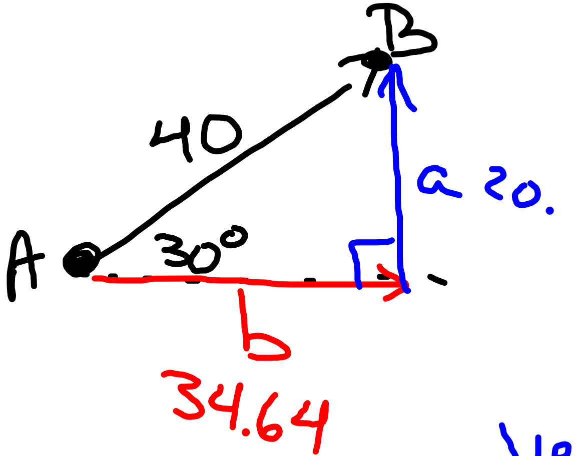
$m\angle\theta = \tan^{-1}\left(\frac{5}{-2}\right)$

$= -68°$

No. We are in Q2.

So $180° - 68° = 112°$

Components of a Vector



horiz comp.

$$\cos 30° = \frac{b}{40}$$

$$b = 40\cos 30$$

$$= 20\sqrt{3}$$

$$= 34.64$$

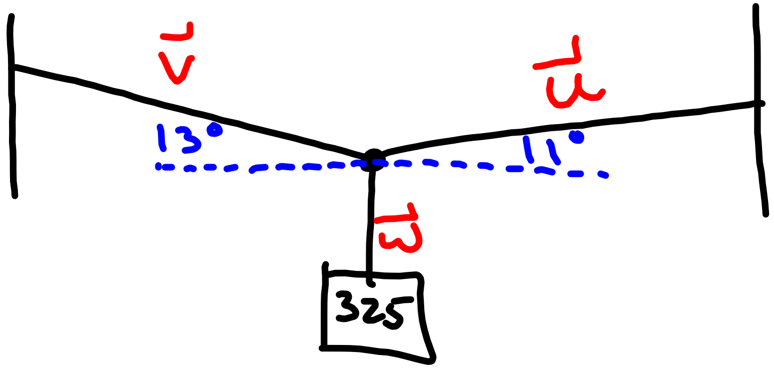
Vert. comp.

$$\sin 30° = \frac{a}{40}$$

$$a = 40\sin 30$$

$$a = 20$$

Ex. 7
p. 393



The horizontal components of $\vec{v}$ & $\vec{u}$ add up to 0. (They cancel). Because the weight doesent move.

The vertical components of $\vec{v}$ & $\vec{u}$ cancel out the vertical component of $\vec{w}$. Again, because the weight doesent move.

Horiz: $\vec{v}$: $\|\vec{v}\|\cos 13^\circ$

$\vec{u}$: $\|\vec{u}\|\cos 11^\circ$

$$\boxed{\|\vec{v}\|\cos 13^\circ = \|\vec{u}\|\cos 11^\circ}$$

Vert: $\vec{v}$: $\|\vec{v}\|\sin 13^\circ$

$\vec{u}$: $\|\vec{u}\|\sin 11^\circ$

$\vec{w}$: 325

$$\boxed{\|\vec{v}\|\sin 13^\circ + \|\vec{u}\|\sin 11^\circ = 325}$$

$$\|\vec{v}\|\cos 13° = \|\vec{u}\|\cos 11°$$

$$\|\vec{v}\|\sin 13° + \|\vec{u}\|\sin 11° = 325$$

$$\|\vec{u}\| = \frac{\|\vec{v}\|\cos 13°}{\cos 11°}$$

$$\|\vec{v}\|\sin 13° + \frac{\|\vec{v}\|\cos 13°}{\cos 11°} \cdot \sin 11° = 325$$

$$.225\|\vec{v}\| + .189\|\vec{v}\| = 325$$

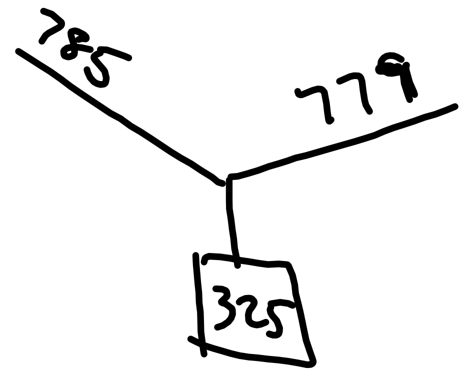
$$.414\|\vec{v}\| = 325$$

$$\|\vec{v}\| = 785 \text{ lbs.}$$

$$\|\vec{u}\| = \frac{\|\vec{v}\| \cos 13^\circ}{\cos 11^\circ} \qquad \|\vec{v}\| = 785 \text{ lbs.}$$

$$\|\vec{u}\| = \frac{785 \cos 13^\circ}{\cos 11^\circ}$$

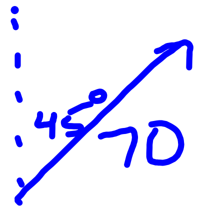
$$= 779 \text{ lbs.}$$



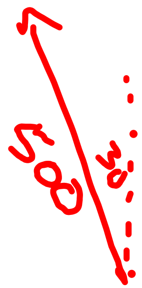
Plane 500 kts @ 330°

Wind 70 kts @ 45°

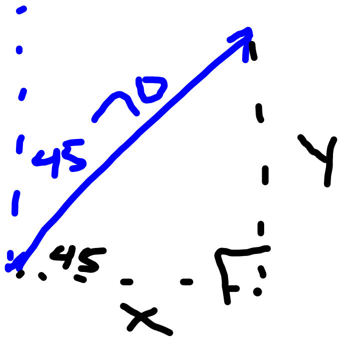
Wind



Plane



Wind



$$\sin 45 = \frac{y}{70}$$

$$y = 70\sin 45^\circ$$

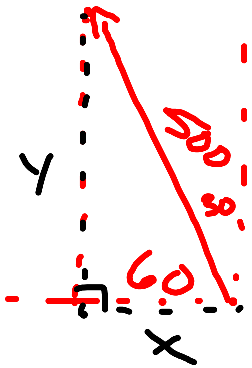
$$y = 49.5$$

And also $x = 49.5$

$$W = 49.5\hat{i} + 49.5\hat{j}$$

$$\vec{W} = 49.5\hat{i} + 49.5\hat{j}$$

Plane



$$\sin 60 = \frac{Y}{500}$$

$$Y = 500 \sin 60$$

$$Y = 433$$

$$\cos 60 = \frac{X}{500}$$

$$X = 500 \cos 60^\circ$$

$$X = 250$$

$$\vec{P} = -250\hat{i} + 433\hat{j}$$

$$\vec{r} = \vec{p} + \vec{w}$$

$$= -250\hat{i} + 433\hat{j}$$

$$+ \quad 49.5\hat{i} + 49.5\hat{j}$$

$$\vec{r} = -200.5\hat{i} + 482.5\hat{j}$$

Now we need to find the speed and direction.

The direction

$$m\angle\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

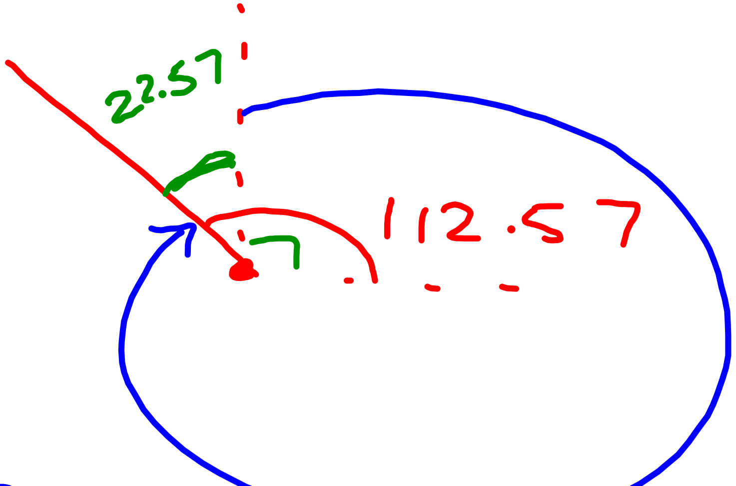
$$= \tan^{-1}\left(\frac{482.5}{-200.5}\right)$$

$$= -67.43^\circ$$

But in Q 2 . . . .

$$180^\circ - 67.43 = 112.57^\circ$$

But we need the compass heading.



$360 - 22.57$

$337.43°$

The speed

$$\sqrt{(-200.5)^2 + 482.5^2}$$

522.5 kts