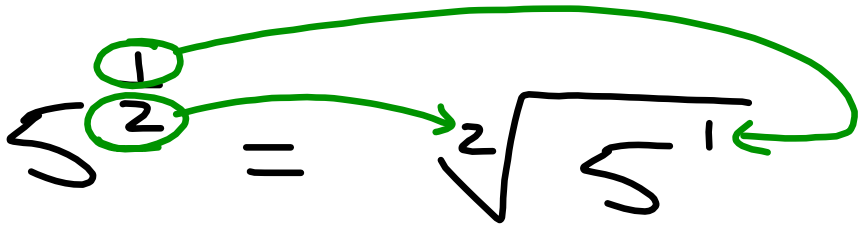
Mar. 28, 2018
Sect. 7-2 & 3
Rational Exponents

Defn



$$7^{\frac{1}{3}} = \sqrt[3]{7}$$

$$z^{\frac{1}{4}} = \sqrt[4]{z}$$

$$y^{\frac{2}{5}} = \sqrt[5]{y^2} \text{ or } \left(\sqrt[5]{y}\right)^2$$

$$x^{\frac{3}{7}} = \sqrt[7]{x^3} \text{ or } \left(\sqrt[7]{x}\right)^3$$

Be Careful

$$2x^{\frac{1}{3}} = 2\sqrt[3]{x}$$

$$(2x)^{\frac{1}{3}} = \sqrt[3]{2x}$$

Simplify (Evaluate)

$$4^{\frac{1}{2}} = \sqrt[2]{4} = 2$$

$$9^{\frac{1}{2}} = \sqrt[2]{9} = 3$$

$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$$

$$9^{\frac{3}{2}} = \sqrt[2]{9^3} \text{ or } \left(\sqrt[2]{9}\right)^3 = 3^3 = 27$$

Properties of Exponents

1. $x^a \cdot x^b = x^{a+b}$

$x^2 \cdot x^3 = x^{2+3} = x^5$

2. $\frac{x^a}{x^b} = x^{a-b}$

3. $(x^a)^b = x^{ab}$

Negative Exponents

$$x^{-a} = \frac{1}{x^a} \neq -x^a$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

Simplify $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

$7^{\frac{1}{2}} \cdot 7^{\frac{1}{2}} = 7^{\frac{1}{2}+\frac{1}{2}} = 7^{\frac{2}{2}} = 7^1 = 7$

or $\sqrt{7} \cdot \sqrt{7} = \sqrt{49} = 7$

$$5^{\frac{1}{3}} \cdot 25^{\frac{1}{3}}$$

$$5^{\frac{1}{3}} \cdot \left(5^2\right)^{\frac{1}{3}} = 5^{\frac{1}{3}} \cdot 5^{\frac{2}{3}}$$

$$= 5^{\frac{1}{3}+\frac{2}{3}}$$

$$= 5^{\frac{3}{3}} = 5^1 = 5$$

or $5^{\frac{1}{3}} \cdot 25^{\frac{1}{3}}$

$\sqrt[3]{5} \cdot \sqrt[3]{25} = \sqrt[3]{125}$

$= 5$

Evaluate $25^{-\frac{5}{2}}$

$$= \frac{1}{25^{\frac{5}{2}}} = \frac{1}{\left(\sqrt{25}\right)^5}$$

$$= \frac{1}{5^5}$$

$$= \frac{1}{3125}$$

$$(-27)^{\frac{2}{3}} = \left(\sqrt[3]{-27}\right)^2$$

$$= (-3)^2$$

$$= 9$$

$$(-27)^{\frac{2}{3}}$$

$$(-3^3)^{\frac{2}{3}} = (-3)^{\frac{6}{3}} = (-3)^2$$

$$= 9$$

$$(243\,a^{-10})^{\frac{2}{5}}$$

$$(243)^{\frac{2}{5}}\,(a^{-10})^{\frac{2}{5}}$$

$$\left(\sqrt[5]{243}\right)^{2}\left(a^{-\frac{20}{5}}\right)$$

$$(3)^{2}\left(a^{-4}\right) \Rightarrow \frac{9}{a^{4}}$$

$$\frac{\sqrt[3]{x^3}}{\sqrt[5]{x^2}} = \frac{x^{\frac{3}{3}}}{x^{\frac{2}{5}}} = \frac{x^1}{x^{\frac{2}{5}}}$$

$$= x^{1-\frac{2}{5}} = x^{\frac{5}{5}-\frac{2}{5}}$$

$$= x^{\frac{3}{5}} = \sqrt[5]{x^3}$$

$$\sqrt{2} \cdot \sqrt[4]{2}$$

$$2^{\frac{1}{2}} \cdot 2^{\frac{1}{4}}$$

$$2^{\frac{1}{2}+\frac{1}{4}} \Rightarrow 2^{\frac{2}{4}+\frac{1}{4}} = 2^{\frac{3}{4}}$$

$$= \sqrt[4]{2^3} = \sqrt[4]{32}$$

$$\sqrt[4]{27} \cdot \sqrt[3]{9} \cdot \sqrt{3}$$

$$\sqrt[4]{3^3} \cdot \sqrt[3]{3^2} \cdot \sqrt[2]{3^1}$$

$$3^{\frac{3}{4}} \cdot 3^{\frac{2}{3}} \cdot 3^{\frac{1}{2}} = 3^{\frac{3}{4}+\frac{2}{3}+\frac{1}{2}}$$

$$= 3^{\frac{9}{12}+\frac{8}{12}+\frac{6}{12}} = 3^{\frac{23}{12}}$$

$$= \sqrt[12]{3^{23}}$$