May 12, 2014

Sect. 7-3

Polar Form of Complex Numbers

$z = a + bi$

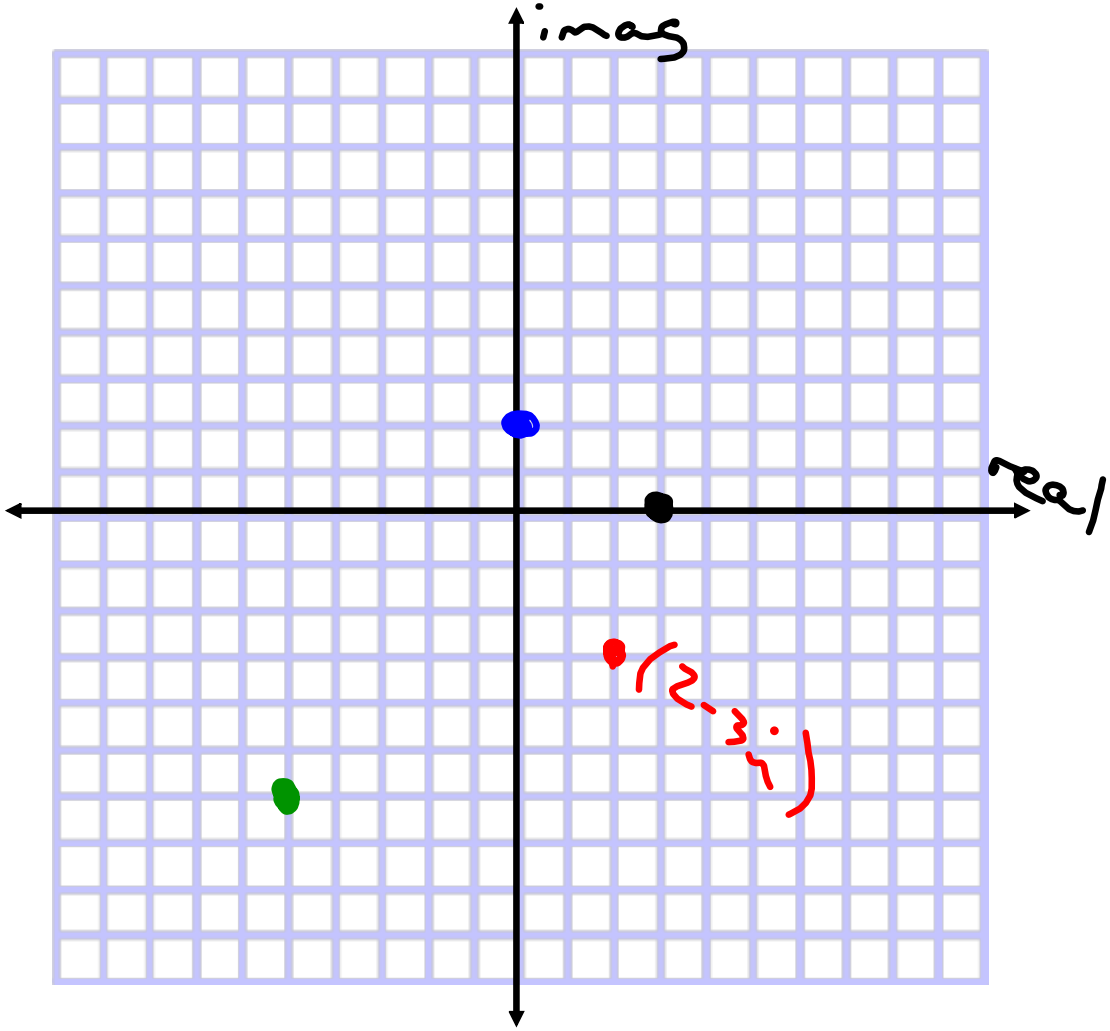
Graphing z

$2 - 3i$

$-5 - 6i$

$2i$

3



Abs Val

$|2| = 2$ $\quad |-4i| = 4$

$|-3-4i| = \sqrt{3^2+4^2} = 5$

This is called the <u>modulus</u> of $a+bi$

Now same thing but with a polar grid.

$$r = \sqrt{x^2+y^2} = \sqrt{a^2+b^2}$$

The radius is the modulus.

$$\theta^{*} = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{b}{a}\right)$$

* Mind the quadrant

$$z = -3 - 4i = (r, \theta)$$

$$\|z\| = r = \sqrt{3^2 + 4^2} = 5$$

$$\theta = \tan^{-1}\left(\frac{-4}{-3}\right) = 53.1^\circ \quad * \text{ Q1}$$

Q3 : $180^\circ + 53.1^\circ = 233.1^\circ$

So $(5, 233.1^\circ)$

$(3, 150^\circ)$

$a = r\cos\theta \qquad b = r\sin\theta$

$a = 3\cos 150^\circ \qquad b = 3\sin 150^\circ$

$a = -2.598 \qquad b = 1.5$

So: $-2.598 + 1.5i$

So . . .

$$-3-4i = 5\cos 233.1^\circ + 5i\sin 233.1^\circ$$

we abbreviate this

$$= 5\,\mathrm{cis}\,233.1^\circ$$

Another abbreviation

$$5e^{i233.1^\circ}$$

In General

$z = a + bi = (r, \theta)$

$r = \sqrt{a^2 + b^2}$ $\quad \theta^* = \tan^{-1}\left(\frac{b}{a}\right)$

* Q

$z = r(\cos\theta + i\sin\theta) = r\,\mathrm{cis}\,\theta$

Rect. to Polar (Trig)

1. Find modulus of z (r)
2. Find θ ($\tan^{-1}$) * Q
3. $z = r\,\text{cis}\,\theta$

$z = -5 + 6i$ Q2

$r = \sqrt{25+36} = \sqrt{61}$

$\theta = \tan^{-1}\left(\frac{6}{-5}\right) = -50.2°$

$\theta = 180 - 50.2° = 129.8°$

$z = \sqrt{61}\,\text{cis}\,129.8°$

Polar (Trig) to Rect.

$z = 6\,\text{cis}\,30°$

$= 6(\cos 30° + i \sin 30°)$

Just do arithmetic.

$6\frac{\sqrt{3}}{2} + 6\left(\frac{1}{2}\right)i$

$z = 3\sqrt{3} + 3i$