May 14, 2014

Sect. 7-4

DeMoivre's Thm

Powers / Roots

Given

z_1

z_2 Add / Subt.

No.

Use a+bi form

$$z_1 = r_1 \operatorname{cis} \theta_1$$

$$z_2 = r_2 \operatorname{cis} \theta_2$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z_1 z_2 z_3 = r_1 r_2 r_3 \operatorname{cis}(\theta_1 + \theta_2 + \theta_3)$$

$$z^n = r^n \operatorname{cis}(n\theta)$$ DeMoivre's Thm

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z = (-1 + i\sqrt{3})^{12}$$

$$z = 2\,\text{cis}\,120^\circ$$

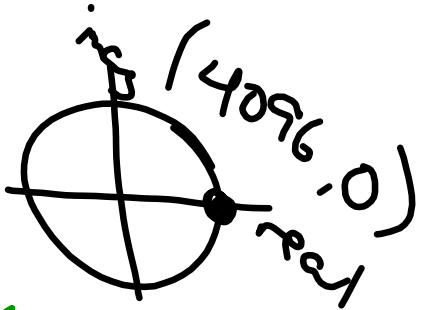
$$z^{12} = 2^{12}\,\text{cis}(12 \cdot 120^\circ)$$

$$= 4096\,\text{cis}\,1440^\circ$$

$$= 4096\,\text{cis}\,0^\circ$$

$$= 4096 + 0i$$

$$= 4096$$



$4096(\cos 0^\circ + i \sin 0^\circ)$

$$z = (-1 + i\sqrt{3})^{\frac{1}{2}}$$

$$z = 2\,\text{cis}\,120^\circ$$

$$z^{\frac{1}{2}} = \sqrt{2}\,\text{cis}\left(\tfrac{1}{2}\cdot 120^\circ\right)$$

$$= \sqrt{2}\,\text{cis}\,60^\circ$$

$$= \sqrt{2}(\cos 60^\circ + i\sin 60^\circ)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$$ * Almost

Solve: $z^3 = -2+2i$

$z = (-2+2i)^{\frac{1}{3}}$

$z = (\sqrt{8}\ \text{cis}\ 135^\circ)^{\frac{1}{3}}$

$z = \sqrt[3]{\sqrt{8}}\ \text{cis}\left(\frac{1}{3}\cdot 135^\circ\right)$

$z_1 = \sqrt[6]{8}\ \text{cis}\ 45^\circ$

$z_2 = \sqrt[6]{8}\ \text{cis}\ 165^\circ$

$z_3 = \sqrt[6]{8}\ \text{cis}\ 285^\circ$