April 4, 2018

Sect. 7-7

Complex Numbers

Defn

Simplify radicals using i

Operations on Complex Numbers

$$\sqrt{9} = 3$$

$$\sqrt{9} = -3$$

$$\sqrt{-9} = ?$$

Define $\sqrt{-1} \equiv i$

$\sqrt{-9} = i\sqrt{9} = 3i$

$\sqrt{-25} = 5i$

$\sqrt{-7} = i\sqrt{7}$

$\sqrt{-12} = i\sqrt{12} = i\sqrt{4\cdot 3} = 2i\sqrt{3}$

Complex Numbers

Defn: A number that has a real part and an imaginary part.

$2 + 3i$

$4 - 5i \quad \Rightarrow \quad a + bi$

$-3 + i$

Operations

Add / Subt.

$$(a+bi) \pm (c+di)$$

$$(a \pm c) + (bi \pm di)$$

$$(2+3i)+(5+7i)$$

$$(2+5)+(3i+7i)$$

$$7+10i$$

$$(4-6i)+(2+3i)$$

$$(4+2)+(-6i+3i)$$

$$6-3i$$

$$(6+2i)-(4+5i)$$

$$(6-4)+(2i-5i)$$

$$2-3i$$

$$(4+7i)-(6-2i)$$

$$(4-6)+(7i-(-2i))$$

$$-2+9i$$

Powers of i

$i^1 = i$

$i^2 = -1 \qquad \left(\sqrt{-1}\right)^2 = (i)^2$

$i^3 = i^2 \cdot i^1 = (-1)(i) = -i$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$

$i^5 = i^4 \cdot i^1 = (1)(i) = i$

$\vdots$

Multiply

$2(3+4i) = 6+8i$

$(2i)(3i) = 6i^2 = 6(-1) = -6$

$(3i)(-6i) = -18i^2 = -18(-1) = 18$

$$(2+3i)(4-5i) \qquad \text{FOIL}$$

$$2(4)+2(-5i)+(3i)(4)+(3i)(-5i)$$

$$8\underbrace{-10i+12i}\overset{+15}{\cancel{-15i^2}}$$

$$23+2i$$

$$(6+i)(4+3i)$$

$$6(4)+6(3i)+(i)(4)+(i)(3i)$$

$$24+\underbrace{18i+4i}+\cancel{3i^2}^{-3}$$

$$21+22i$$

$$(2-3i)^2$$

$$(2-3i)(2-3i)$$

$$4 \underbrace{-6i-6i} + \cancel{9i^2} - 9$$

$$-5-12i$$

$$(2+3i)(2-3i)$$

$$2(2)+2(-3i)+(3i)(2)+(3i)(-3i)$$

$$4\underbrace{-6i+6i}\overset{+9}{\cancel{-9i^2}}$$

$$13+0i$$

$$13$$

Complex Conjugates

Comp. Conj.

$2+3i \Rightarrow 2-3i$

$-6+i \Rightarrow -6-i$

$4-5i \Rightarrow 4+5i$

Division

$$\frac{3}{(2+7i)} \cdot \frac{(2-7i)}{(2-7i)}$$

$$\frac{6-21i}{4 \underbrace{-14i+14i} \underset{+49}{\cancel{-49i^2}}}$$

$$\frac{6-21i}{53} = \frac{6}{53} - \frac{21}{53}i$$

$$\frac{(5-3i)}{(2+i)} \cdot \frac{(2-i)}{(2-i)}$$

$$\frac{10-5i-6i+\cancel{3i^2}^{\,-3}}{4 \cancel{-i^2} +1}$$

$$\frac{7-11i}{5} = \frac{7}{5}-\frac{11}{5}i$$