

**Example 12** Translating and Solving a Compound Inequality

The sum of a number and 4 is between  $-5$  and  $12$ . Find all such numbers.

**Solution:**

Let  $x$  represent a number.

$$-5 < x + 4 < 12$$

Translate the inequality.

$$-5 - 4 < x + 4 - 4 < 12 - 4$$

Subtract 4 from all three parts of the inequality.

$$-9 < x < 8$$

The number may be any real number between  $-9$  and  $8$ :  $\{x \mid -9 < x < 8\}$ .

**Skill Practice**

17. The sum of twice a number and 11 is between 21 and 31. Find all such numbers.

**Answer**

17. Any real number between 5 and 10:  $\{n \mid 5 < n < 10\}$

**Section 1.5** Practice Exercises

Boost your GRADE at  
ALEKS.com!

**ALEKS**  
Assessment and Learning in Knowledge Spaces

- Practice Problems
- Self-Tests
- NetTutor

- e-Professors
- Videos

**Study Skills Exercises**

- Which activities might you try when working in a study group to help you learn and understand the material?
  - Quiz one another by asking one another questions.
  - Practice teaching one another.
  - Share and compare class notes.
  - Support and encourage one another.
  - Work together on exercises and sample problems.
- Define the key terms.
  - a. Compound inequality
  - b. Intersection
  - c. Union

**Review Exercises**

For Exercises 3–6, solve the linear inequality. Write the solution in interval notation.

3.  $-6u + 8 > 2$

4.  $2 - 3z \geq -4$

5.  $-12 \leq \frac{3}{4}p$

6.  $5 > \frac{1}{3}w$

**Concept 1: Union and Intersection of Sets**

7. Given:  $M = \{-3, -1, 1, 3, 5\}$  and  
 $N = \{-4, -3, -2, -1, 0\}$ . (See Example 1.)

List the elements of the following sets:

- a.  $M \cap N$       b.  $M \cup N$

8. Given:  $P = \{a, b, c, d, e, f, g, h, i\}$  and  
 $Q = \{a, e, i, o, u\}$ .

List the elements of the following sets.

- a.  $P \cap Q$       b.  $P \cup Q$

For Exercises 9–20, refer to the sets  $A, B, C,$  and  $D$ . Determine the union or intersection as indicated. Express the answer in interval notation, if possible. (See Example 2.)

$$A = \{x|x < -4\}, B = \{x|x > 2\}, C = \{x|x \geq -7\}, D = \{x|0 \leq x < 5\}$$

- |                |                |                |
|----------------|----------------|----------------|
| 9. $A \cap C$  | 10. $B \cap C$ | 11. $A \cup B$ |
| 12. $A \cup D$ | 13. $A \cap B$ | 14. $A \cap D$ |
| 15. $B \cup C$ | 16. $B \cup D$ | 17. $C \cap D$ |
| 18. $B \cap D$ | 19. $C \cup D$ | 20. $A \cup C$ |

For Exercises 21–26, find the intersection and union of sets as indicated. Write the answers in interval notation. (See Example 3.)

- |                                       |                                    |   |
|---------------------------------------|------------------------------------|---|
| 21. a. $(-2, 5) \cap [-1, \infty)$    | 22. a. $(-\infty, 4) \cap [-1, 5)$ | 23. a. $\left(-\frac{5}{2}, 3\right) \cap \left(-1, \frac{9}{2}\right)$ |
| b. $(-2, 5) \cup [-1, \infty)$        | b. $(-\infty, 4) \cup [-1, 5)$     | b. $\left(-\frac{5}{2}, 3\right) \cup \left(-1, \frac{9}{2}\right)$     |
| 24. a. $(-3.4, 1.6) \cap (-2.2, 4.1)$ | 25. a. $(-4, 5] \cap (0, 2]$       | 26. a. $[-1, 5) \cap (0, 3)$  |
| b. $(-3.4, 1.6) \cup (-2.2, 4.1)$     | b. $(-4, 5] \cup (0, 2]$           | b. $[-1, 5) \cup (0, 3)$  |

**Concept 2: Solving Compound Inequalities: And**

For Exercises 27–36, solve the inequality and graph the solution. Write the answer in interval notation. (See Examples 4–6.)

- |  |  |
|--|--|
| 27. $y - 7 \geq -9$ and $y + 2 \leq 5$<br>_____                              | 28. $a + 6 > -2$ and $5a < 30$<br>_____                                    |
| 29. $2t + 7 < 19$ and $5t + 13 > 28$<br>_____                                | 30. $5p + 2p \geq -21$ and $-9p + 3p \geq -24$<br>_____                    |
| 31. $2.1k - 1.1 \leq 0.6k + 1.9$ and<br>$0.3k - 1.1 < -0.1k + 0.9$<br>_____  | 32. $0.6w + 0.1 > 0.3w - 1.1$ and<br>$2.3w + 1.5 \geq 0.3w + 6.5$<br>_____ |
| 33. $\frac{2}{3}(2p - 1) \geq 10$ and $\frac{4}{5}(3p + 4) \geq 20$<br>_____ | 34. $\frac{5}{2}(a + 2) < -6$ and $\frac{3}{4}(a - 2) < 1$<br>_____        |
| 35. $-2 < -x - 12$ and $-14 < 5(x - 3) + 6x$<br>_____                        | 36. $-8 \geq -3y - 2$ and $3(y - 7) + 16 > 4y$<br>_____                    |

**Concept 3: Solving Inequalities of the Form  $a < x < b$**

- |   |  |
|---|--|
| 37. Write $-4 \leq t < \frac{3}{4}$ as two separate inequalities. | 38. Write $-2.8 < y \leq 15$ as two separate inequalities. |
| 39. Explain why $6 < x < 2$ has no solution.                      | 40. Explain why $4 < t < 1$ has no solution.               |
| 41. Explain why $-5 > y > -2$ has no solution.                    | 42. Explain why $-3 > w > -1$ has no solution.             |

For Exercises 43–54, solve the inequality and graph the solution set. Write the answer in interval notation. (See Examples 7–8.)

43.  $0 \leq 2b - 5 < 9$



44.  $-6 < 3k - 9 \leq 0$



45.  $-1 < \frac{a}{6} \leq 1$



46.  $-3 \leq \frac{1}{2}x < 0$



47.  $-\frac{2}{3} < \frac{y-4}{-6} < \frac{1}{3}$



48.  $\frac{1}{3} > \frac{t-4}{-3} > -2$



49.  $5 \leq -3x - 2 \leq 8$



50.  $-1 < -2x + 4 \leq 5$



51.  $12 > 6x + 3 \geq 0$



52.  $-4 \geq 2x - 5 > -7$



53.  $-0.2 < 2.6 + 7t < 4$



54.  $-1.5 < 0.1x \leq 8.1$



#### Concept 4: Solving Compound Inequalities: Or

For Exercises 55–64, solve the inequality and graph the solution set. Write the answer in interval notation. (See Examples 9–10.)

55.  $2y - 1 \geq 3$  or  $y < -2$



56.  $x < 0$  or  $3x + 1 \geq 7$



57.  $1 > 6z - 8$  or  $8z - 6 \leq 10$



58.  $22 > 4t - 10$  or  $7 > 2t - 5$



59.  $5(x - 1) \geq -5$  or  $5 - x \leq 11$



60.  $-p + 7 \geq 10$  or  $3(p - 1) \leq 12$



61.  $\frac{5}{3}v \leq 5$  or  $-v - 6 < 1$



62.  $\frac{3}{8}u + 1 > 0$  or  $-2u \geq -4$



63.  $0.5w + 5 < 2.5w - 4$  or  $0.3w \leq -0.1w - 1.6$



64.  $1.25a + 3 \leq 0.5a - 6$  or  $2.5a - 1 \geq 9 - 1.5a$



#### Mixed Exercises

For Exercises 65–74, solve the inequality. Write the answer in interval notation.

65. a.  $3x - 5 < 19$  and  $-2x + 3 < 23$

b.  $3x - 5 < 19$  or  $-2x + 3 < 23$

66. a.  $0.5(6x + 8) > 0.8x - 7$  and  $4(x + 1) < 7.2$

b.  $0.5(6x + 8) > 0.8x - 7$  or  $4(x + 1) < 7.2$

67. a.  $8x - 4 \geq 6.4$  or  $0.3(x + 6) \leq -0.6$

b.  $8x - 4 \geq 6.4$  and  $0.3(x + 6) \leq -0.6$

68. a.  $-2r + 4 \leq -8$  or  $3r + 5 \leq 8$

b.  $-2r + 4 \leq -8$  and  $3r + 5 \leq 8$

69.  $-4 \leq \frac{2 - 4x}{3} < 8$

70.  $-1 < \frac{3 - x}{2} \leq 0$